RESPONSE OF RAILWAY WAGONS TO RANDOM TRACK UNEVENNESS AND LOW JOINTS

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SHASHIKANT LOKRAS

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CERTIFICATE

This is to certify that the work "Response of Railway Wagons to Random Track Unevenness and Low Joints" has been carried out under my supervision and has not been submitted elsewhere for a degree.

N.C. Nigam

Date: 18.7.81

N.C. NIGAM
Professor
Department of Aeronautical Engineering
Indian Institute of Technology, Kanpur,
INDIA

A C K N O W E E D G E M E N T S

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CONTENTS

SYMBOLS

A	$((-\alpha_i)^2/v^2) + \Omega^2$
a	amplitude
c	Roughness constant, 1/h damping constant
a_o, a_n, b_n	Fourier constants
$E(\ )$	Expected value
$F(S)$	Contact force
g	Acceleration due to gravity
$H(\Omega)$	Impulsive response function
h	Depth of pulse
h_n	Depth of n_{th} pulse
J	$\sqrt{-1}$
K	Stiffness constant
K_{ZZ}	Covariance function of Z
$K_{\ddot{Z}\ddot{Z}}$	Covariance function of $\ddot{Z}$
$K_{\ddot{Y}\ddot{Y}}$	Covariance function of $\ddot{Y}$
L	Distance between two consecutive pulses
l	length of pulse
l_n	length of n_{th} pulse
M	Sprung mass
m	Unsprung mass
M_i	$2\,\Phi_i^2\,(\alpha_i + \zeta\omega_o)$
N	Number of pulses passed
n	number of pulse considered
$p(l)$	Probability density function of l.
r	Dimensionless ratio, $2\pi V/\lambda\omega_o$, $V\pi/l\omega_o$
S	Distance
t	Time

u	α_i/v, $1/L$
x_{ih}	Homogenous solution
X_{inh}	Non-homogenous solution
$Y(S)$	Base displacement
$\ddot{Y}(S)$	Base acceleration
$Z(S)$	Absolute displacement
$\ddot{Z}(S)$	Absolute acceleration
α_i	Eigen values
β_i	Constants
ϕ_i	Eigen vectors
γ	$\tan^{-1} 2\zeta r$, constant
ζ	Damping ratio
Ω	Spatial frequency, π/l
λ	wave length in sinusoidal track profile
$\Phi_{yy(\Omega)}$	Power spectral density of continuous unevenness
σ	R.m.s. value
ω_0	Natural frequency
$\Phi_{ZZ(\Omega)}$	Power spectral density of Z
$(\cdot)$	Differentiation with respect to time.
$(')$	Differentiation with respect to distance.

CHAPTER I

INTRODUCTION

A vehicle moving in contact with the ground follows the ground profile. The ground unevenness therefore acts as a base excitation for the vehicle through the points of contact. The nature of the ground induced excitation is determined by the characteristics of the ground unevenness, type of contact and vehicle speed. Most vehicle operates on prepared tracks such as roads, rails, runways etc. This thesis deals with the response of railway wagons, coaches etc., to the vertical unevenness of the rail track.

A rail track consists of fishplated rail joints and is supported on sleepers and ballast. The sleepers hold the rail in correct alignment and transmits the track load to the ballast over a wide area. A rail joint is the weakest part of the track and a study of the effect of these joints on vertical response and track forces is very important from the point of view of design of vehicle and track. A very common feature of the joints is the hogging of the rails with the passage of vehicle. When hogging exceeds a certain limit, the joints are called low joints. As heavy maintenance work is required for these joints, welded joints are now being provided over several weld lengths on high speed track. However, fishplated joints still exist to allow thermal expansion of the rails.

The unevenness of the track can be considered to be of two types:

1. Continuous unevenness due to irregularities of the rail profile, and
2. Discrete unevenness due to hogging of the rails at the joints.

The study of vibration effects on the vehicle due to above two types of unevenness are significant in respect of i) design of vehicle structure, ii) passenger comfort, iii) controllability of the vehicle, and iv) design of the track. The vibration of the vehicle induces dynamic loads on the vehicle and track structure. The ground induced loads may cause fracture or fatigue failure of structural components.

Literature Survey

The response of vehicles to continuous track unevenness has been studied by several investigators, only a few investigations are reported in open literature for discrete low joints. Radford (4) has investigated the vertical force between rail wheel and rail at dipped rail joint. By attempting a deterministic solution, he has shown that the impulsive force produced due to passage of heavily loaded vehicles on such irregularities increased linearly with the speed. By using symmetrical rail joints, he has also shown that the reduction in unsprung mass causes the reduction in contact force which is most important for the deterioration of the joint.

In reference (14), Fryba has given various methods for calculating the vibration effects of base when a moving load passes over it.

In reference (9), Yadav has extensively calculated the response of different vehicle models over random track irregularities by taking unevenness as a homogenous process.

Analytical Investigations

The analytical investigation reported in thesis covers three aspects, i) mathematical model of the vehicle, ii) model of track unevenness, iii) solution of governing equations to obtain response statistics.

Single degree of freedom model with linearised spring and dampers is investigated. It is assumed that vehicles moves with constant velocity. Model of track unevenness consists of continuous irregularities of the vertical rail profile plus unevenness of the joints. The governing equations are solved by using convolution integral approach.

The parameters investigated in the thesis are: i) maximum value of vertical displacement, ii) maximum value of vertical acceleration, and iii) maximum and minimum value of the contact force. The upper limit to the response as well as response acceleration is important for passenger comfort and drivers ability to control the vehicle. This sets a limit to the vehicle velocity and makes the dynamic response a constraint in vehicle under-carriage design. The upper limit to

the contact force may be important to the vehicle control system as well as for stress calculations in structural components of the track and vehicle. The lower value of contact force is important for derailment studies.

A chapterwise summary of the contents of the thesis is given below:

Chapter 2 gives a description of mathematical models of the vehicle and track unevenness. The derivation of the equation of motion and their solution by convolution integral have been discussed.

In Chapter 3, response parameters for harmonic excitation are determined. The analysis of such a theoretical excitation to the vehicle is useful as it will form a basis for Fourier analysis considered later in Chapter 4. Since response to harmonic excitation is easy to calculate, Fourier analysis is an important tool for calculating responses of different type of excitations.

Chapter 4 describes the response of vehicle to the periodically occurring low joints. Response parameters are determined by using two methods, (a) single pulse excitation: in this analysis response of the vehicle for single joint excitation is calculated. Some suitable shape of the pulse (joint) is assumed. Then response for series of pulses are obtained by superimposing response of each pulse, when length and depth of successive pulses are, (i) identical, (ii) different. While considering single pulse excitation, it is of interest to calculate response parameters near the

starting point of successive pulses. Such an information later on helps in determining the response of vehicle due to the excitation of series of pulses. (b) Fourier analysis: Fourier analysis is an important tool for solving different type of excitations. Since these pulses occur periodically, equivalent Fourier series can be considered for the excitation. In the Fourier analysis response for each harmonic component is calculated and then response of vehicle due to the excitation of Fourier series is obtained by superimposing response of each harmonic component. In Chapter 3, response parameters for harmonic excitation have been already calculated.

In Chapter 5, length and depth of pulses which were deterministic in previous chapter are taken as random. Such an analysis is realistic in calculating fatigue damage of the joint and vehicle components. A fatigue failure is the result of cumulative damage that arises when response of structure to external excitation fluctuates. Uniform probability distributions have been assumed for random variables. First and second order statistics of response parameters for the n_{th} pulse are determined and then by using superposition principle, second order statistics for series of n pulses are obtained.

In Chapter 6, first and second order statistics of the response parameters to continuous unevenness are calculated. The continuous unevenness has been treated as a homogenous random process with zero mean. Later on continuous unevenness

is superimposed upon discrete unevenness due to random low joints. Second order statistics of the response parameters for total unevenness is then obtained by assuming the two processes are uncorrelated.

Summary and conclusions are described in Chapter 7.

Following are some of the important conclusions drawn from the analysis.

(A) Response of vehicle to discrete unevenness shows that:

(i) Peak values of maximum absolute displacement, acceleration and contact force occur for a particular value of dimensionless ratio r =(1.1 in our analysis). This critical value of r depends upon velocity, length of pulse, natural frequency and damping coefficient. Typical peak values for $r = 1.1$, natural frequency $\omega_0 = 13.808$ rad/sec and pulse length = 1.5m are as follows.

Peak value of absolute displacement is about 5.3 times depth of pulse, absolute acceleration is 0.5 times acceleration due to gravity and contact force is 1.45 times the total weight of the vehicle.

(ii) Maximum values of response parameters vary linearly with $2\alpha v$ (where 2α is the depth angle = 4 times ratio of depth to the length of pulse and v is the velocity), keeping 2α and r as fixed.

(iii) A limiting velocity can be found above which vehicle loses its contact to the ground. This velocity is 80 km/hr for r = 1.1.

(iv) (a) For a velocity less than 50 km/hr, interaction, effect of only previous pulse is significant.

(b) for a velocity ling between 50 to 100 km/hr, effect of at least 3 previous pulses should be considered.

(c) For a velocity greater than 100 km/hr, effect of atleast 5 pulses is necessary in summation.

(v) For a velocity of 60 km/hr and a variation of pulse length from 1.5 meters to 7.5 meters, range of $(F_{max}/(M + m)g)$ is from 1.27 to 1.78, while for $(F_{min}/(M+m)g)$ is 0.42 to 0.95. F_{max} and F_{min} are max. and min. contact force and $(M + m)g$ is the total weight of the vehicle.

Probability of getting $F_{max}/(M+m)g$ (also known as dynamic load factor (DLF) greater than 1.50 is very high (0.8), while probability of getting DLF greater than 1.75 is only 0.1.

(vi) Assuming the design DLF as 1.77 at a velocity of 60 km/hr, the probability of failure for one km. length of track is as high as 0.96. It increases as we decrease the value of design DLF.

(B) Response of vehicle to continuous unevenness shows that

(i) R.m.s. value of acceleration and contact force increase linearly at high velocity. Typical values at a

velocity of 150 km/hr are $\sigma_{\ddot{Z}}$ = 0.27 times acceleration due to gravity and σ_F = 0.24 times the weight of vehicle. Results vary much depend upon chosen value of roughness constant.

(C) Response of vehicle to total unevenness (discrete and continuous) shows that,

i) Expected value of total response process is same as that of the expected value of response process due to discrete unevenness.

ii) R.m.s. values of total response process increases with velocity.

CHAPTER 2

MATHEMATICAL MODELS OF TRACK AND VEHICLE

INTRODUCTION

A railway vehicle is supported over wheels attached to the undercarriage system. The vehicle undercarriage system consists of springs and shock absorbers, supported on wheels. Railway wagons generally use laminated springs. A sketch ~~diagram~~ of the vehicle is shown in Fig.2.1.

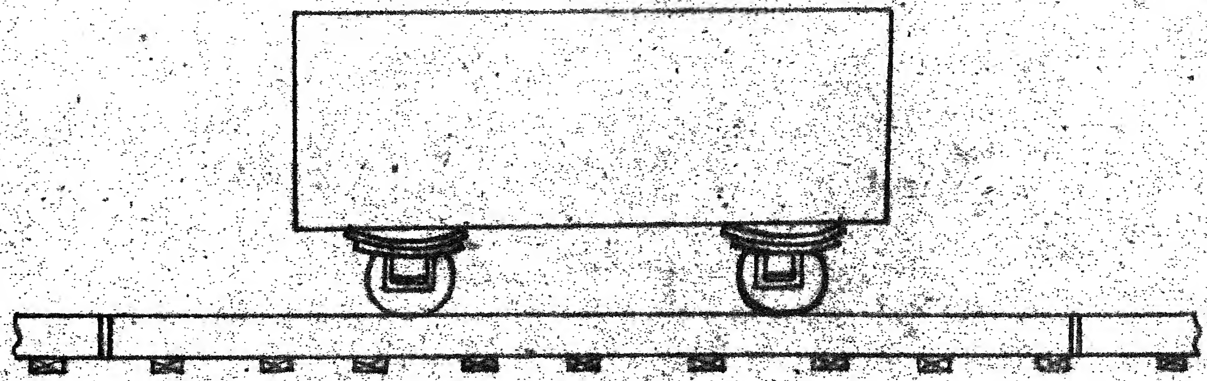
A rail track as shown in Fig.2.2, consists of fishplated rail joints and is supported by sleepers. The sleepers hold the rail in correct alignment and transmitts the track load to the ballast.

This chapter describes the construction of the idealized models for vehicle and track unevenness. Equations of motion are derived and transformed to a system of first order uncoupled equation of motions.

DESCRIPTION OF MODELS

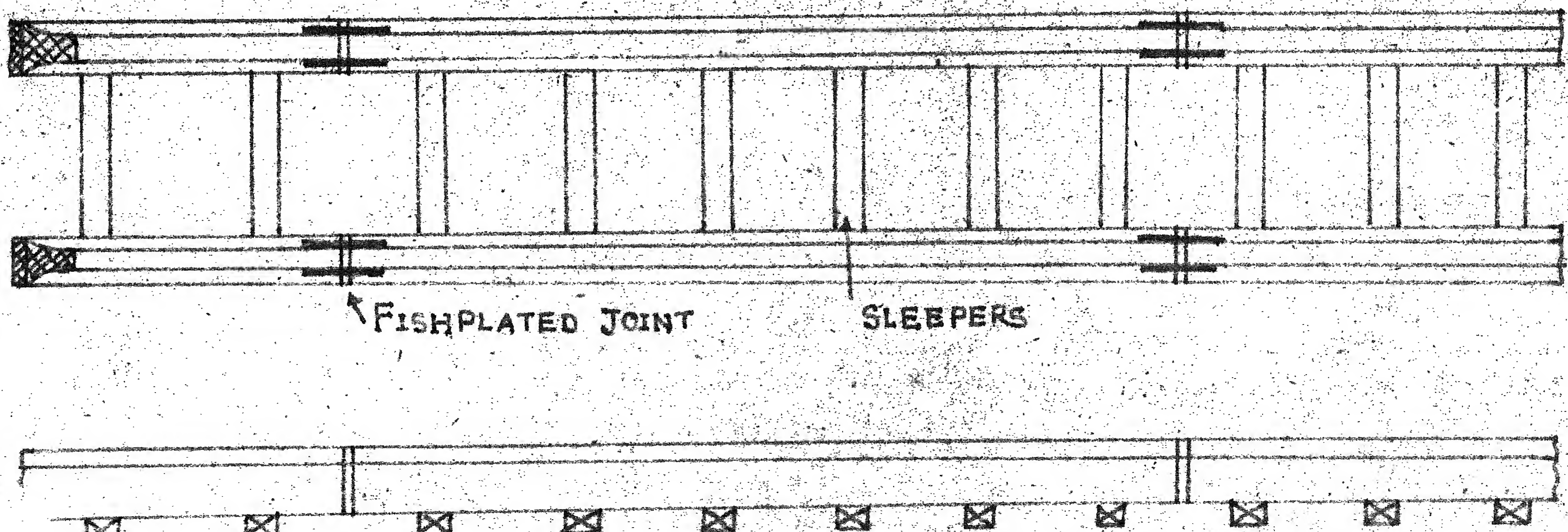
2.1 Vehicle Model

The vehicle as described consists of several degrees of freedom and the analysis of such a complicated model is quite, difficult. Hence to simplify, it is assumed that there is only one degrees of freedom in vertical direction and the whole vehicle mass (called sprung mass) is mounted on the linearized spring and damper. The mass of wheel, axle etc.,



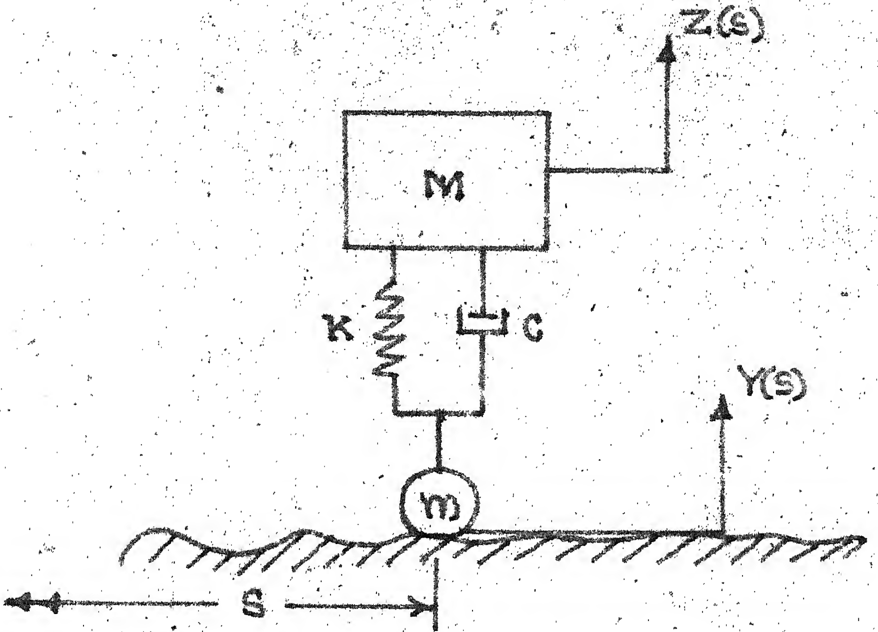
SKETCH DIAGRAM OF A VEHICLE

FIG. 2·1



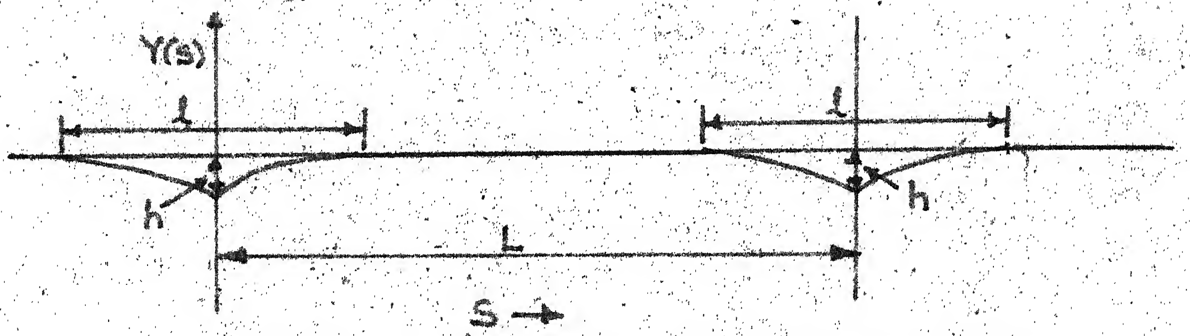
SKETCH DIAGRAM OF A RAILWAY TRACK

FIG. 2·2



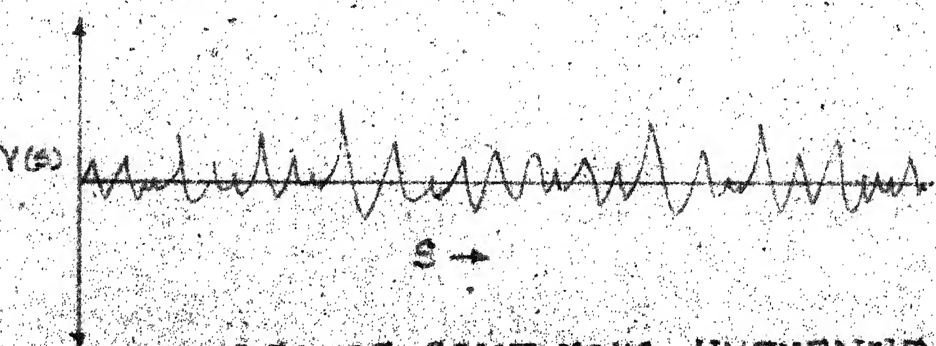
VEHICLE MODEL

FIG. 2.3



MODEL OF DESCRETE UNEVENNESS

FIG. 2.4



MODEL OF CONTINOUS UNEVENNESS

FIG. 2.5

(called unsprung mass) is concentrated on wheel itself. Fig.2.3 shows the linear model with single degree of freedom. In the model M and m are sprung and unsprung masses respectively. K and c are linear spring and damping coefficients. Z(S) and Y(S) are absolute displacement of sprung and unsprung masses respectively.

Vehicle Motion:

Vehicle is assumed to be moving with constant velocity v given by

$$S = vt$$

where S and t are distance and time respectively.

Parameter Investigated:

Interest lies in the calculation of the following parameters.

i) Maximum value of absolute displacement Z(S) of M.

ii) Maximum value of absolute acceleration $\ddot{Z}(S)$ of M

iii) Maximum and minimum value of contact force F(S), where F(S) is defined as

$$F(S) = M\ddot{Z}(S) + m\ddot{Y}(S) + (M + m)\ g$$

where (.) denotes differentiation with respect to time, g is the acceleration due to gravity.

2.2 Models of Track Unevenness

The unevenness of the track can be considered to be of two types,

i) Discrete unevenness due to hogging of the rails at the joints.

ii) continuous unevenness due to the irregularity of the rail profile.

2.2.1 Discrete Unevenness

The shape of the discrete unevenness of the joints can be assumed to be of the following form (fig.2.4).

$$Y(S) = -h\left(1 + \sin \frac{\pi}{l} (S - nL)\right) \text{ when } nL - \frac{l}{2} \leq S \leq nL$$

$$= -h\left(1 - \sin \frac{\pi}{l} (S - nL)\right) \text{ when } nL \leq S \leq nL + \frac{l}{2}$$

$$= 0 \text{ otherwise}$$

where l = length of pulse

L = length between two consecutive pulses

n = number of pulse considered

h = maximum depression or depth of the rails.

When a vehicle passes over such a pulse, the response parameters are calculated by two methods.

(i) Single pulse Excitation:

In this analysis, response parameters are calculated for single pulse excitation only. Response for series of pulses is then obtained by superimposing response for each pulse.

(ii) Fourier analysis:

Since the occurrences of the pulses are periodic. Fourier analysis can be applied here. Fourier analysis is easy to apply for any pulse shape. The equivalent Fourier

series can be written as

$$Y(S) = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{2n\pi S}{L} + b_n \sin \frac{2n\pi S}{L}$$

where a_o, a_n and b_n are Fourier coefficients and can be obtained by following integrals:

$$a_o = \frac{2}{L} \int_0^L Y(S)\, dS$$

$$a_n = \frac{2}{L} \int_0^L Y(S) \cos \frac{2n\pi S}{L}\, dS$$

$$b_n = \frac{2}{L} \int_0^L Y(S) \sin \frac{2n\pi S}{L}\, dS$$

Since Fourier series is a combination of sine and cosine terms, the analysis of sinusoidal excitation is necessary for calculating response of vehicle due to each harmonic component.

The shape of the sinusoidal excitation can be assumed to be of the form

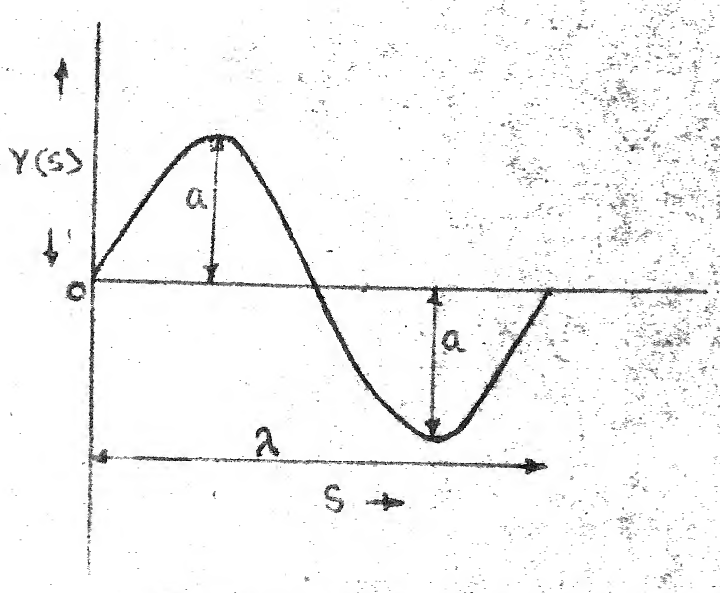
$$Y(S) = a \sin \frac{2\pi S}{\lambda}$$

where λ is the wave length and a is the amplitude of undulations.

In Chapter 3, sinusoidal excitation of the above form is considered and response parameters are calculated.

<u>Continuous Unevenness:</u> (Fig.2.5)

Continuous unevenness due to irregularities of the rail profile has been assumed to be a homogenous random process



SINUSOIDAL TRACK PROFILE MODEL

FIG. 2.6

with zero mean and spectral density given by:

$$\Phi_{YY}(\Omega) = \frac{C}{\Omega^2 + \gamma^2} \quad |\Omega| \leq \Omega_o$$

$$= 0 \text{ otherwise.}$$

Where C and γ are constants. Ω is spatial frequency.

The constant C is given by

$$C = \frac{\sigma^2 \gamma}{\pi}$$

where σ is the R.m.s. value of unevenness.

2.3 Equation of Motion

As shown in Fig.2.3 the inertia force due to mass M is $M\ddot{Z}$, where $\ddot{Z}$ is the absolute acceleration of mass M. The base is given a displacement Y(S). The spring force is due to the deformation of spring and is equal to $K(Z - Y)$. The damping force is proportional to relative velocity $(\dot{Z} - \dot{Y})$ and is equal to the $C(\dot{Z} - \dot{Y})$. Summing up the forces gives

$$M\ddot{Z} + C(\dot{Z} - \dot{Y}) + K(Z - Y) = 0$$

or $$M\ddot{Z} + C\dot{Z} + KZ = C\dot{Y} + KY \qquad 2.1$$

where (.) denotes differentiation with respect to time dividing throughout by M we get

$$\ddot{Z} + \frac{C}{M}\dot{Z} + \frac{K}{M}Z = \frac{C}{M}\dot{Y} + \frac{K}{M}Y$$

writing $\frac{C}{M} = 2\zeta\omega_o$ and $\frac{K}{M} = \omega_o^2$

we get

$$\ddot{Z} + 2\zeta\omega_o\dot{Z} + \omega_o^2 Z = 2\zeta\omega_o\dot{Y} + \omega_o^2 Y \qquad 2.2$$

Where ζ is the damping factor and ω_0 is the natural frequency.

For constant velocity run

$$\frac{dS}{dt} = v$$

hence equation 2.2 can be written in the space domain as

$$v^2 Z'' + 2\zeta\omega_0 v Z' + \omega_0^2 Z = 2\zeta\omega_0 v Y' + \omega_0^2 Y \qquad 2.3$$

where (') denotes differentiation with respect to distance S.

Equation 2.2 can be expressed as a system of first order equations (see Appendix I) in the form

$$\frac{dx_i}{dt} - \alpha_i x_i = \frac{\phi_i}{M_i}\left\{2\zeta\omega_0 \dot{Y} + \omega_0^2 Y\right\} \qquad 2.4$$

$$i = 1, 2$$

where $\alpha_i = (-\zeta \pm\sqrt{\zeta^2 - 1})$

$$M_i = 2\phi_i^2(\zeta\omega_0 + \alpha_i)$$

and ϕ_i is the eigen vector of the dynamic matrix (Appendix I).

above equation can also be written in space domain as

$$\frac{dx_i}{dS}\cdot\frac{dS}{dt} - \alpha_i x_i = \frac{\phi_i}{M_i}\left\{2\zeta\omega_0\frac{dY}{dS}\cdot\frac{dS}{dt} + \omega_0^2 Y\right\} \qquad 2.5$$

$$i = 1, 2$$

For constant velocity run

$$\frac{dS}{dt} = v.$$

Substituting this value equation 2.5 becomes

$$v x_i' - \alpha_i x_i = \frac{\phi_i}{M_i}\left\{2\zeta\omega_0 Y' v + \omega_0^2 Y\right\} \qquad 2.6$$

$$i = 1, 2$$

2.4 Solution of Equation of Motion

Equation 2.6 is a constant coefficient first order linear differential equation. The homogenous solution is represented by

$$x_i = e^{A_i S} \qquad i = 1,2$$

where A_i are constants.

Substituting in homogenous part of 2.6, we get

$$vA_i = \alpha_i$$

or $$A_i = \frac{\alpha_i}{v}$$

hence $$x_{ih} = \beta_i \, e^{\frac{\alpha_i S}{v}} \qquad i = 1,2 \qquad 2.7$$

h stands for homogenous,

β_i are constants to be determined from initial conditions.

Non-homogenous solution can be written in terms of convolution integral

$$x_{inh} = \int_0^S h_i(S - p)\, P_i(p)\, dp \qquad 2.8$$

where $$P_i(p) = \frac{\Phi_i}{M_i} \left(2\zeta\omega_0 \frac{dY}{dp} \cdot \frac{dp}{dt} + \omega_0^2 Y \right)$$

$$h_i(S-p) = \frac{e^{\frac{\alpha_i}{v}(S-p)}}{v}$$

also for constant velocity motion $\frac{dp}{dt} = v$

substituting these values, we get

$$x_{inh} = \int_0^S \frac{e^{\frac{\alpha_i}{v}(S-p)}}{v} \; \frac{\Phi_i}{M_i} \left(2\zeta\omega_0 v \frac{dY}{dp} + \omega_0^2 Y \right) dp \qquad 2.8$$

Absolute displacement Z(S) is the summation of homogenous and non-homogenous part.

$$Z(S) = \sum_{1=1}^{2} \Phi_i x_{ih} + \sum_{i=1}^{2} \Phi_i \, x_{inh}$$

$$= \sum_{i=1}^{2} \Phi_i \, \beta_i e^{\frac{\alpha_i S}{v}} + \sum_{i=1}^{2} \frac{\Phi_i^{\,2}}{M_i v} \int_{o}^{S} e^{\frac{\alpha_i}{v}(S-p)} \left\{ 2 \zeta \omega_o v \frac{dY}{dp} + \omega_o^2 Y \right\} dp \qquad 2.10$$

CHAPTER 3

RESPONSE OF VEHICLE TO SINUSOIDAL TRACK PROFILE

INTRODUCTION

Analysis of harmonic excitation to the vehicle is necessary as it will form a basis for Fourier analysis considered later in Chapter 4. Fourier series is the combination of sine and cosine terms. Hence for calculating the response of vehicle for Fourier series, response of vehicle due to each harmonic component is calculated and then superimposed.

3.1 Solution of Equation of Motion

As shown in Fig. 2.6, sinusoidal track profile is represented by

$$Y = a \sin \frac{2\pi S}{\lambda}$$

substituting the values of Y and Y' in equation 2.9 and changing variable from S to p, we get non-homogenous solution

$$x_{inh} = \int_0^S \frac{e^{\frac{\alpha_i}{v}(S-p)}}{v} \frac{\phi_i}{M_i} \left(\frac{4\zeta\omega_o va}{\lambda} \cos \frac{2\pi p}{\lambda} + \omega_o^2 a \sin \frac{2\pi p}{\lambda} \right) dp$$

$$i = 1,2$$

$$= a \frac{\phi_i}{M_i} e^{\frac{\alpha_i S}{v}} \int_0^S \left(\frac{4\zeta\omega_o v\pi}{\lambda} \cos \frac{2\pi p}{\lambda} + \omega_o^2 \sin \frac{2\pi p}{\lambda} \right) e^{-\frac{\alpha_i p}{v}} dp$$

$$= \frac{a\,\Phi_i}{M_i}\;\frac{e^{\frac{\alpha_i S}{v}}}{v}\left(\left(\frac{4\zeta\omega_o v\pi}{\lambda^2}\right)^2 + \omega_o^4\right)^{\frac{1}{2}}$$

$$\int_0^S e^{-\frac{\alpha_i p}{v}} \sin\left(\frac{2\pi p}{\lambda} + \gamma\right) dp$$

where $\gamma = \tan^{-1} \frac{4\zeta\omega_o v\pi}{\omega_o^2}$

$$x_{inh} = \frac{a\,e^{\frac{\alpha_i S}{v}}\Phi_i}{M_i v}\left(\left(\frac{4\zeta\omega_o v\pi}{\lambda^2}\right)^2 + \omega_o^4\right)^{\frac{1}{2}} \cdot$$

$$\left(\frac{e^{-\frac{\alpha_i p}{v}}}{\left(\frac{-\alpha_i}{v}\right)^2 + \left(\frac{2\pi}{\lambda}\right)^2}\right)\left(-\frac{\alpha_i}{v}\sin\left(\frac{2\pi p}{\lambda}+\gamma\right) - \frac{2\pi}{\lambda}\cos\left(\frac{2\pi p}{\lambda}+\gamma\right)\right)\Bigg|_0^S$$

$$= \frac{a\,\Phi_i}{M_i v}\left(\left(\frac{4\zeta\pi v\omega_o}{\lambda^2}\right)^2 + \omega_o^4\right)^{1/2} \cdot \frac{1}{\left(\frac{-\alpha_i}{v^2}\right)^2 + \left(\frac{2\pi}{\lambda^2}\right)^2}\left(-\frac{\alpha_i}{v}\sin\right.$$

$$\left.\left(\frac{2\pi S}{\lambda}+\gamma\right) - \frac{2\pi}{\lambda}\cos\left(\frac{2\pi S}{\lambda}+\gamma\right) - e^{\frac{\alpha_i S}{v}}\left(-\frac{\alpha_i}{v}\sin\gamma - \frac{2\pi}{\lambda}\cos\gamma\right)\right)$$

Homogenous solution as obtained in 2.7

$$x_{ih} = \beta_i e^{\frac{\alpha_i S}{v}} \qquad 3.2$$

hence

(i) <u>Absolute Displacement:</u> $Z(S)$

$$Z(S) = \sum_{i=1}^{2} \Phi_i x_{ih} + \sum_{i=1}^{2} \Phi_i x_{inh}$$

$$= \sum_{i=1}^{2} \phi_i \beta_i e^{\frac{\alpha_i S}{v}} + \sum_{i=1}^{2} \frac{a \phi_i^2}{M_i v} \left(\frac{(4\zeta\omega_0 v\pi)^2}{\lambda^2} + \omega_0^4 \right)^{\frac{1}{2}}$$

$$\cdot \frac{1}{\left(\frac{-\alpha_i}{v}\right)^2 + \left(\frac{2\pi}{\lambda}\right)^2} \left(-\frac{2\pi}{\lambda} \cos\left(\frac{2\pi S}{\lambda} + \gamma\right) - \frac{\alpha_i}{v} \sin\left(\frac{2\pi S}{\lambda} + \gamma\right) \right.$$

$$\left. - e^{\frac{\alpha_i S}{v}} \left\{ -\frac{\alpha_i}{v} \sin\gamma - \frac{2\pi}{\lambda} \cos\gamma \right\} \right) \qquad 3.3$$

ii) <u>Absolute acceleration</u>: $\ddot{Z}(S)$

$$\ddot{Z}(S) = \sum_{i=1}^{2} \phi_i \beta_i \alpha_i^2 e^{\frac{\alpha_i S}{v}} + \sum_{i=1}^{2} \frac{a \phi_i^2 v}{M_i} \left\{ \frac{(4\zeta\pi v\omega_0)^2}{\lambda^2} + \omega_0^4 \right\}^{\frac{1}{2}}$$

$$\cdot \frac{1}{\frac{(-\alpha_i)^2}{v^2} + \frac{(2\pi)^2}{\lambda^2}} \left\{ \frac{(2\pi)^2}{\lambda^2} \left(\frac{\alpha_i}{v} \sin\left(\frac{2\pi S}{\lambda} + \gamma\right) + \frac{2\pi}{\lambda} \cos \right.\right.$$

$$\left.\left. \left(\frac{2\pi S}{\lambda} + \gamma\right) \right) - \frac{(\alpha_i)^2}{(v)^2} \left(-\frac{\alpha_i}{v} \sin\gamma - \frac{2\pi}{\lambda} \cos\gamma \right) \left(e^{\frac{\alpha_i S}{v}} \right) \right\} \qquad 3.4$$

iii) <u>Contact Force</u> : F(S)

$$F(S) = M\ddot{Z}(S) + m\ddot{Y}(S) + (M+m)g$$

$$= M \left(\sum_{i=1}^{2} \phi_i \beta_i \alpha_i^2 e^{\frac{\alpha_i S}{v}} + \sum_{i=1}^{2} \frac{a \phi_i^2 v}{M_i} \left(\right.\right.$$

$$\left(\frac{(4\zeta\omega_0 v\pi)^2}{\lambda^2} + \omega_0^4 \right)^{\frac{1}{2}}$$

$$\frac{1}{\left(\frac{-\alpha_i}{v}\right)^2 + \left(\frac{2\pi}{\lambda}\right)^2} \left(\left(\frac{2\pi}{\lambda}\right)^2 \left\{ \frac{\alpha_i}{v} \sin\left(\frac{2\pi S}{\lambda} + \gamma\right) + \frac{2\pi}{\lambda} \cos\left(\frac{2\pi S}{\lambda} + \gamma\right) \right\} \right.$$

$$-\frac{(\alpha_i)^2}{(v)^2}\, e^{\frac{\alpha_i S}{v}} \left\{ -\frac{\alpha_i}{v} \sin r - \frac{2\pi}{\lambda} \cos r \right\} \Big) \Big) \Big)$$

$$-m\left(\frac{2\pi v}{\lambda}\right)^2 a \sin \frac{2\pi S}{\lambda} + (M+m)g \qquad 3.5$$

3.2 Results

Results presented in this section are for bogie CR- 28251, the data for which is given in the table 3.1 (Page 23) other data used in calculation are-

i) Amplitude (a) = 0.1
ii) wave length (λ) = 10 meters
iii) Damping ratio (ζ) = 0.2
iv) Range of velocity = 0 to 200 km/hr

Following observations may be made from the results (Fig. 3.1 to 3.7).

1. Dimensionless ratio $r(=2\pi v/\lambda\omega_0)$ is a very important parameter for the analysis, as it relates velocity to the wave length. Thus for any fixed velocity and natural frequency, increase in r implies decrease in wave length and vice versa. Similarly for any fixed λ and natural frequency increase in r implies, increase in velocity and vice versa.

2. Ratio of the maximum value of absolute displacement to the amplitude (a) has been plotted in fig. 3.1 against r. It is seen that ratio increases very sharply at low velocities, peak value at r = 1.0 and then decreases very sharply upto r = 2.0. This critical value of r depends upon

damping ratio (ζ). Peak value is about two times amplitude.

3. Ratio of the maximum value of absolute acceleration to the acceleration due to gravity has been plotted against r, in Fig.3.2 Ratio increases very sharply at low velocities, peak value at r = 1.0 and then decreases slowly. Peak value of ratio is about 5.25 times acceleration due to gravity.

4. Ratio of the maximum value of contact force to the total weight of the vehicle has been plotted against r in fig 3.3 Ratio increases very sharply at low velocities peak at r = 1.0 and then decreases slowly. Peak value is about 6 times the total weight of the vehicle.

5. In all the figures, it is seen that a particular value of r give all the peak values. This critical value of r depends upon natural frequency, velocity and damping ratio and is important for the design of vehicle undercarriage system.

6. Ratio of the minimum value of the contact force to the total weight of the vehicle has been plotted against r in figure 3.4. Ratio decreases very sharply upto r = 1.0 and then increases slowly. It is seen that at r = .62, contact force is zero which implies loss of contact of the vehicle to the ground. Since ve icle losses its contact, governing equation of motions will not be applicable after that velocity.

7. Ratios of absolute displacement to the amplitude, absolute acceleration to the acceleration due to gravity and contact force to the total weight of the vehicle have been plotted against S/λ in figures 3.5 to 3.7. S is the distance along the wave length λ ; value of r is 0.2. It is seen for that for sinusoidal exictations, ratios also vary sinusoidaly (as expected). Also maximum positive and maximum negative values are same. Thus it is concluded from the analysis that a value of dimensionless ratio r gives peak values of displacement, acceleration and contact force. Results very much depend, upon chosen value of amplitude. Also a velocity can be found above which vehicle losses its contact to the ground.

TABLE 3.1

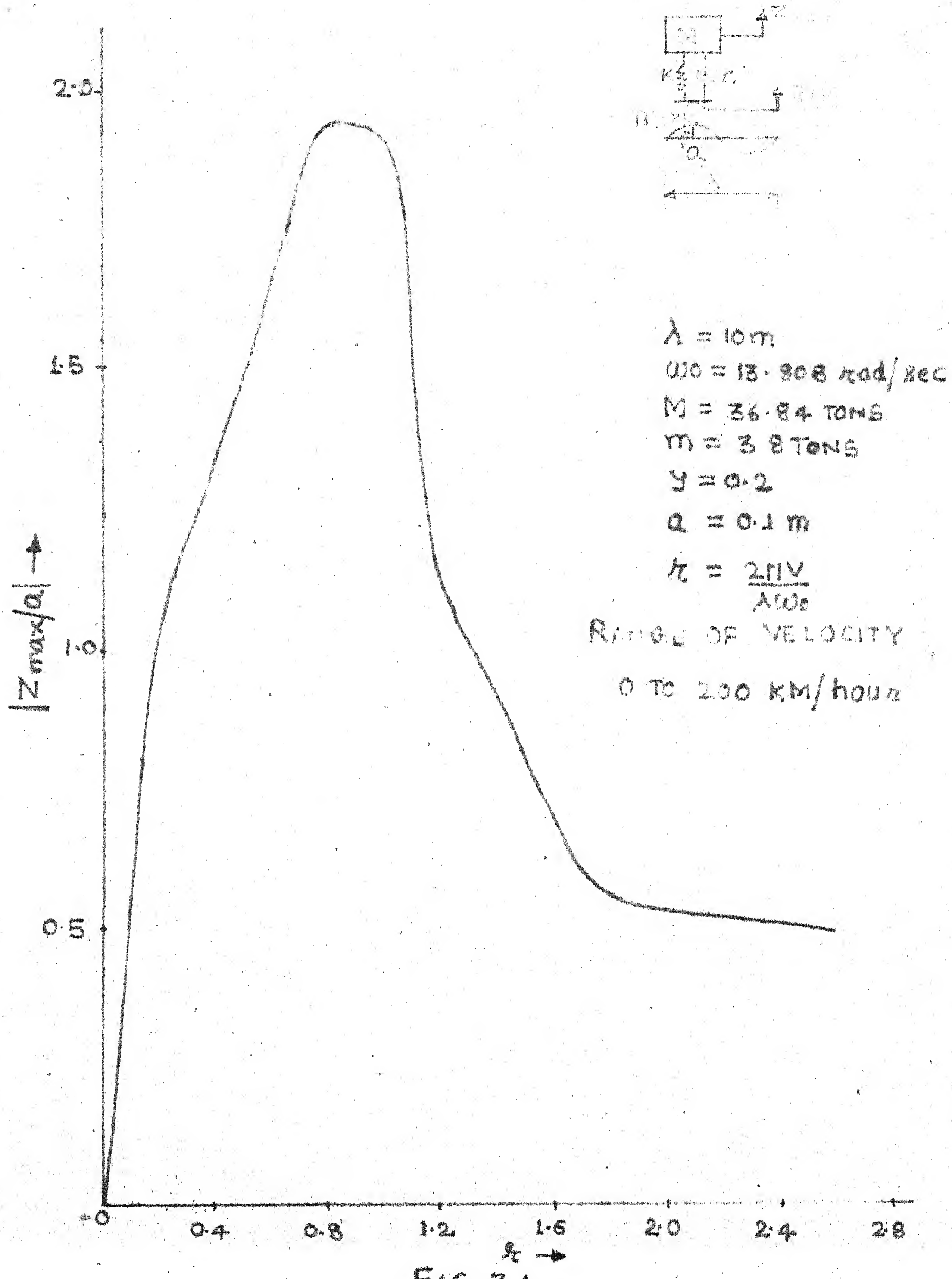
VEHICLE CR - 28251

		Load in tons
1	Tare	= 11.0
2	Gross load	= 40.64
3	Carrying capacity	= 29.64
4	Axle load	= 40.6412 = 20.32
5.	Unsprung weight(m)	= 3.8
6	Sprung weight on each wheel	
	a) Under tare	= 1.80
	b) Under gross	= 9.21
7.	Weight on each axle	
	a) Under tare	= 1.8 x 2= 3.6
	b) Under gross	= 9.21x2= 18.42
8.	Total sprung weight (M)	= (40.64 - 3.80)
		= 36.84
9.	Average spring stiffness	= 0.179 tons/mm
	There are four springs on two axles, each axle having two springs. All springs are assumed to be in parallel combination.	
	Total spring stiffness	= 4 x.179
		= 0.716 tons/mm
10.	The natural frequency of the vehicle in determined as follows:	

$$\omega_0^2 = 0.716 \times 1000/\ (36.84/9.81)$$

$$= 190.66$$

or $\omega_0 = 13.808$ rad/sec.



2·0
1·5
1·0
0·5
0
0·4
0·8
1·2
1·6
2·0
2·4
2·8
$|Z_{max}/a| \rightarrow$
$r \rightarrow$
λ = 10m
ω_0 = 13·908 rad/sec
M = 36·84 TONS
m = 3 8 TONS
y = 0·2
a = 0·1 m
$r = \frac{2\pi V}{\lambda \omega_0}$
RANGE OF VELOCITY
0 TO 200 KM/hour

FIG 3·1

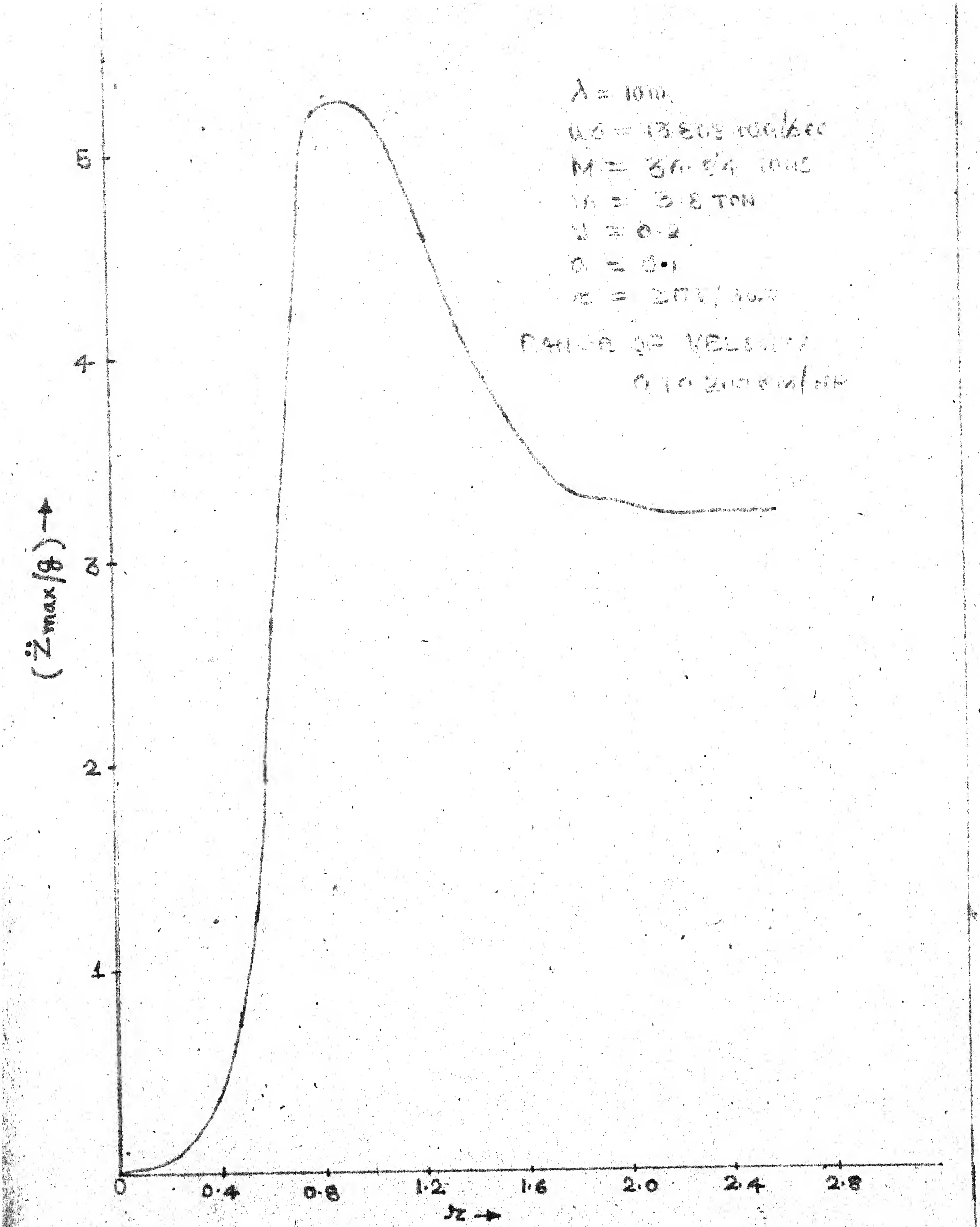

λ = 10m
5
4
3
2
1
0
0.4
0.8
1.2
1.6
2.0
2.4
2.8
$(\ddot{Z}_{max}/g) \rightarrow$
$x \rightarrow$

FIG 3.2

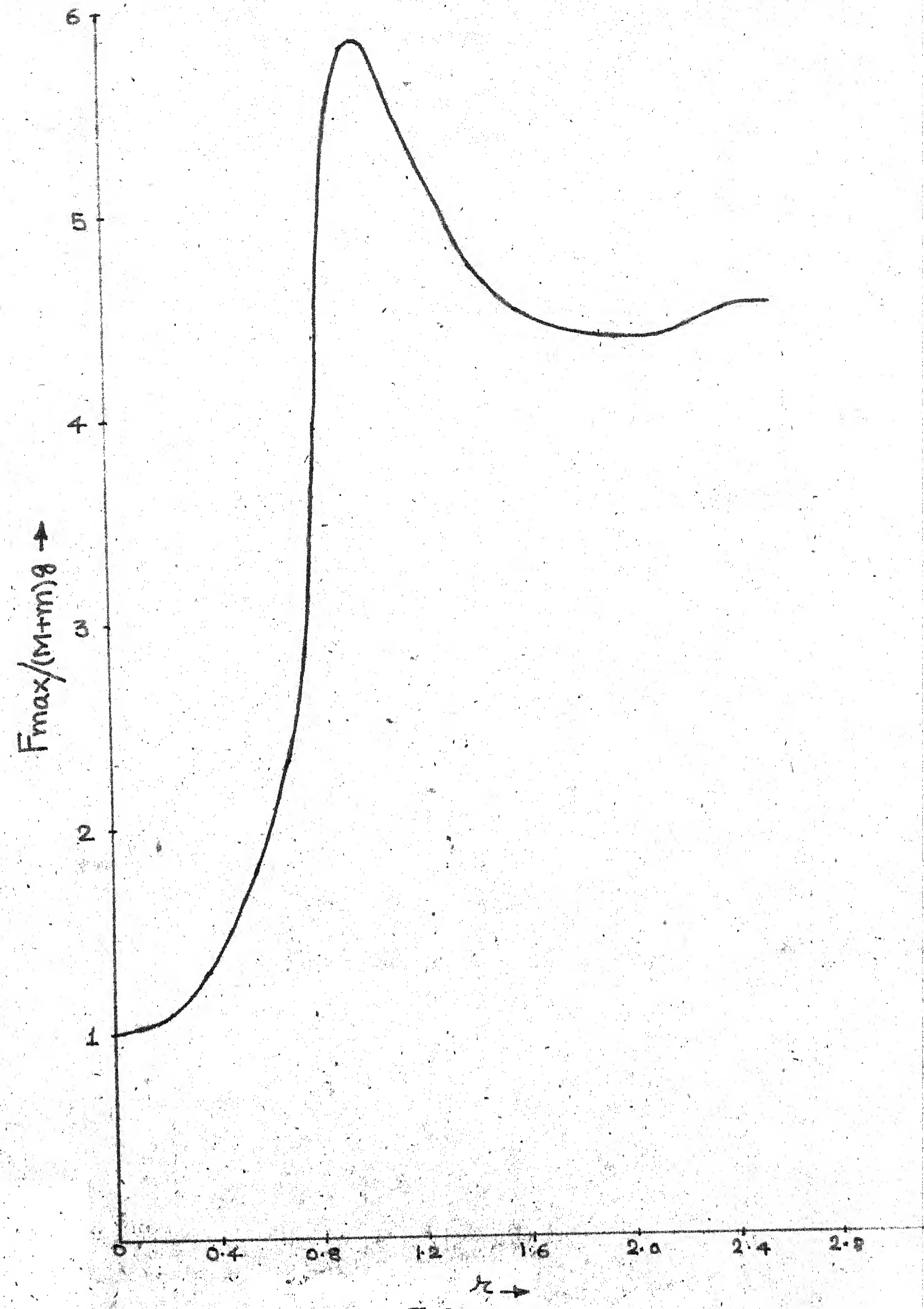


FIG. 3.3

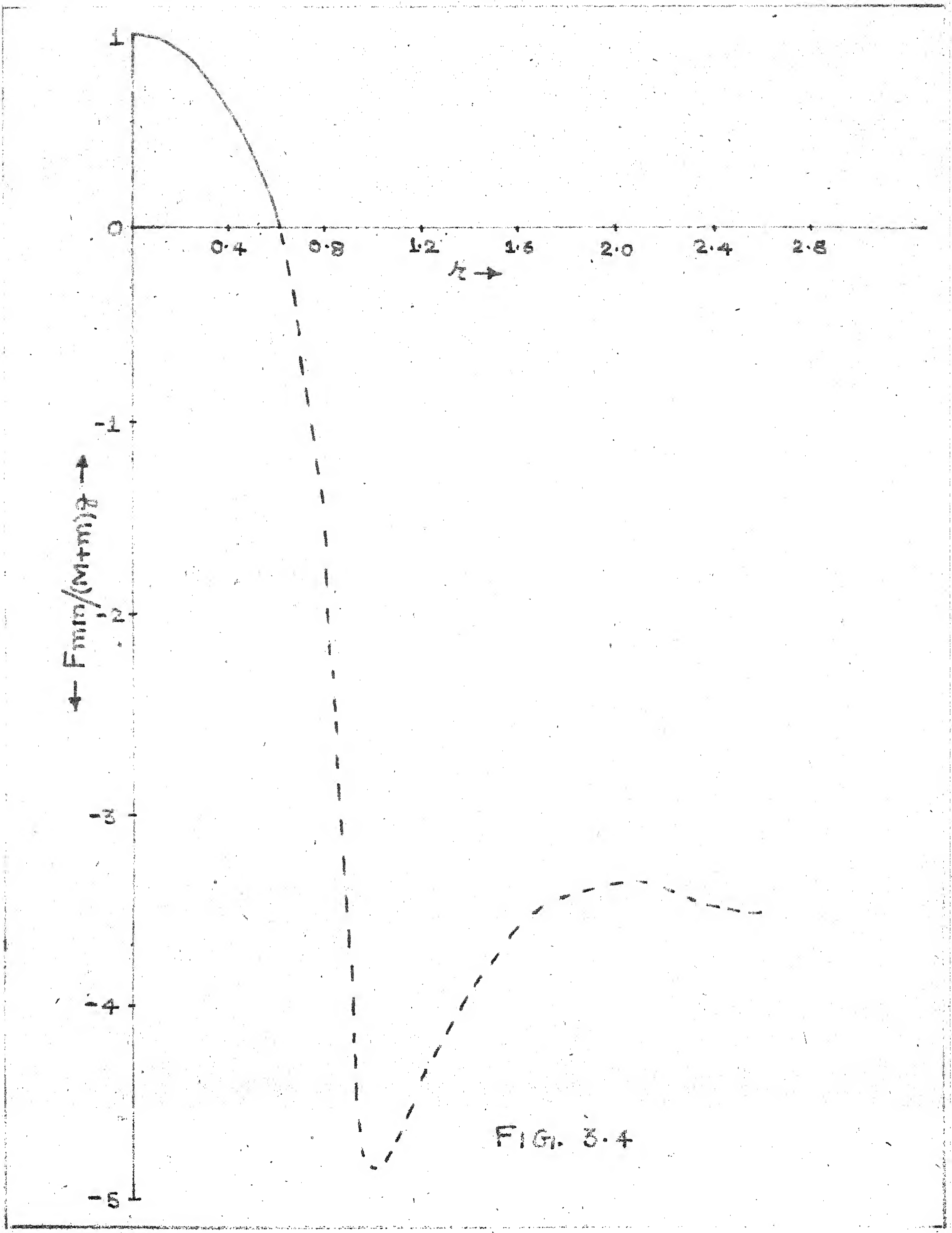
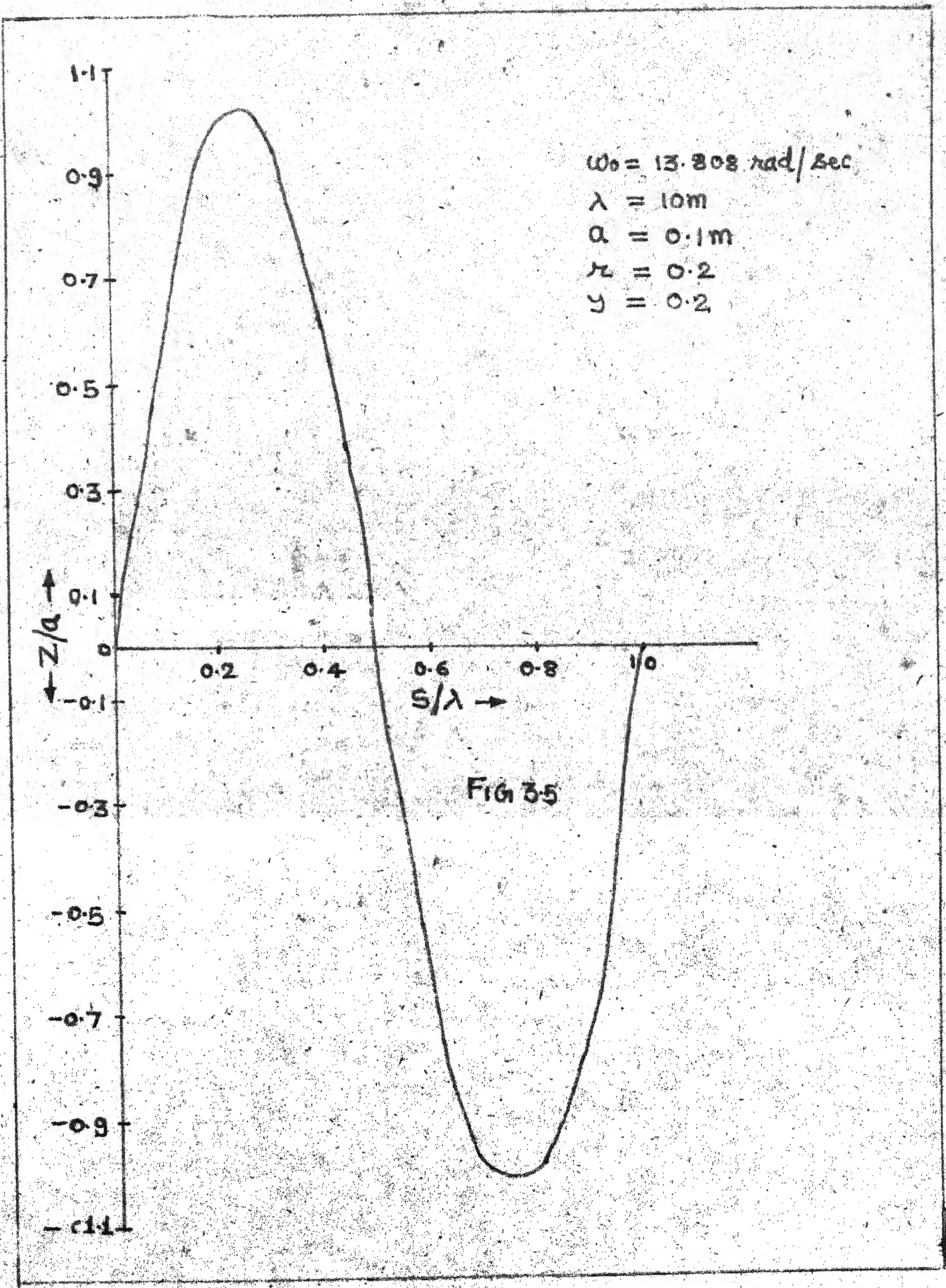
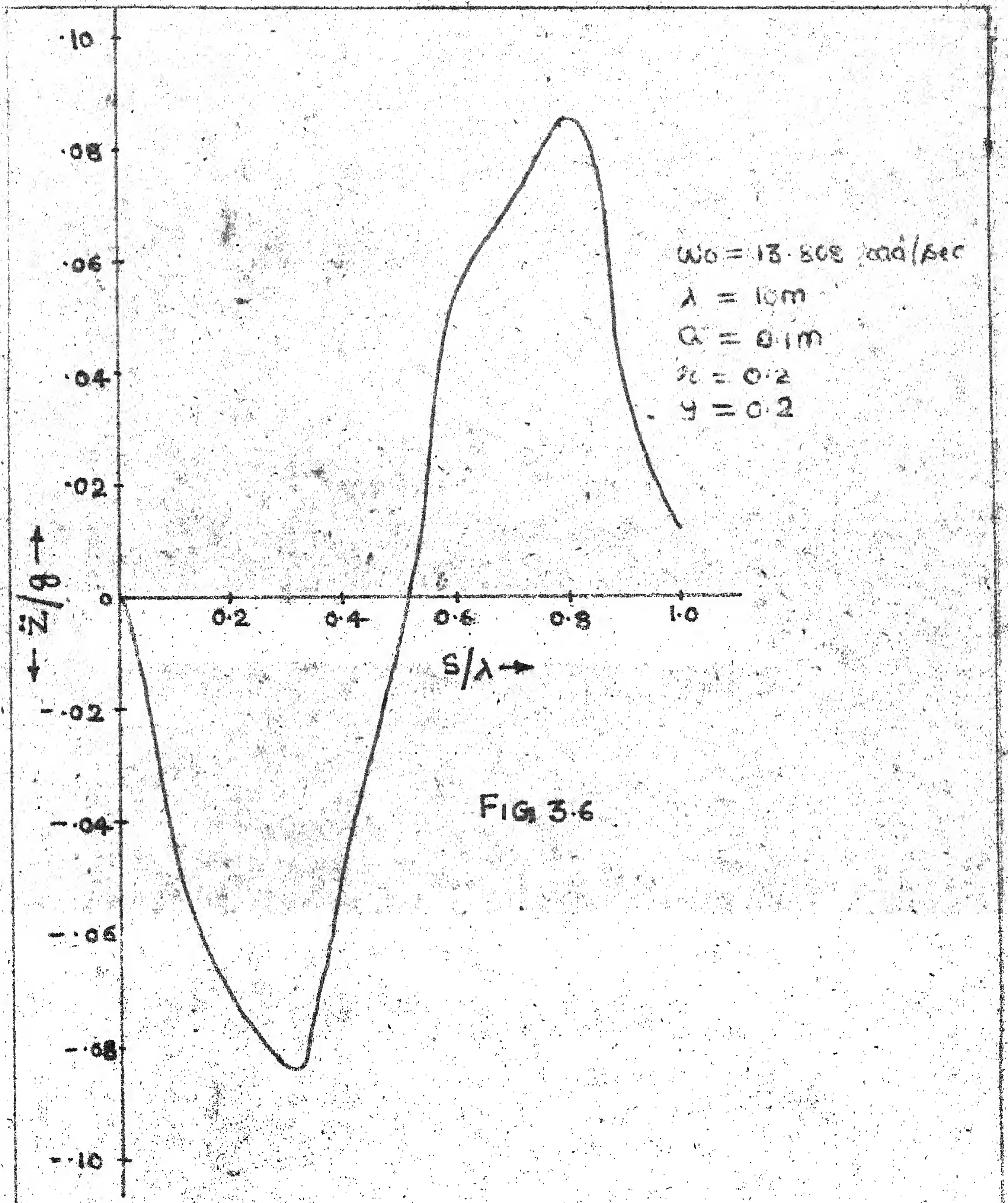

1
0
-1
-2
-3
-4
-5
0·4
0·8
1·2
1·6
2·0
2·4
2·8
$k \rightarrow$
$\leftarrow F_{min}/(M+m)g \rightarrow$

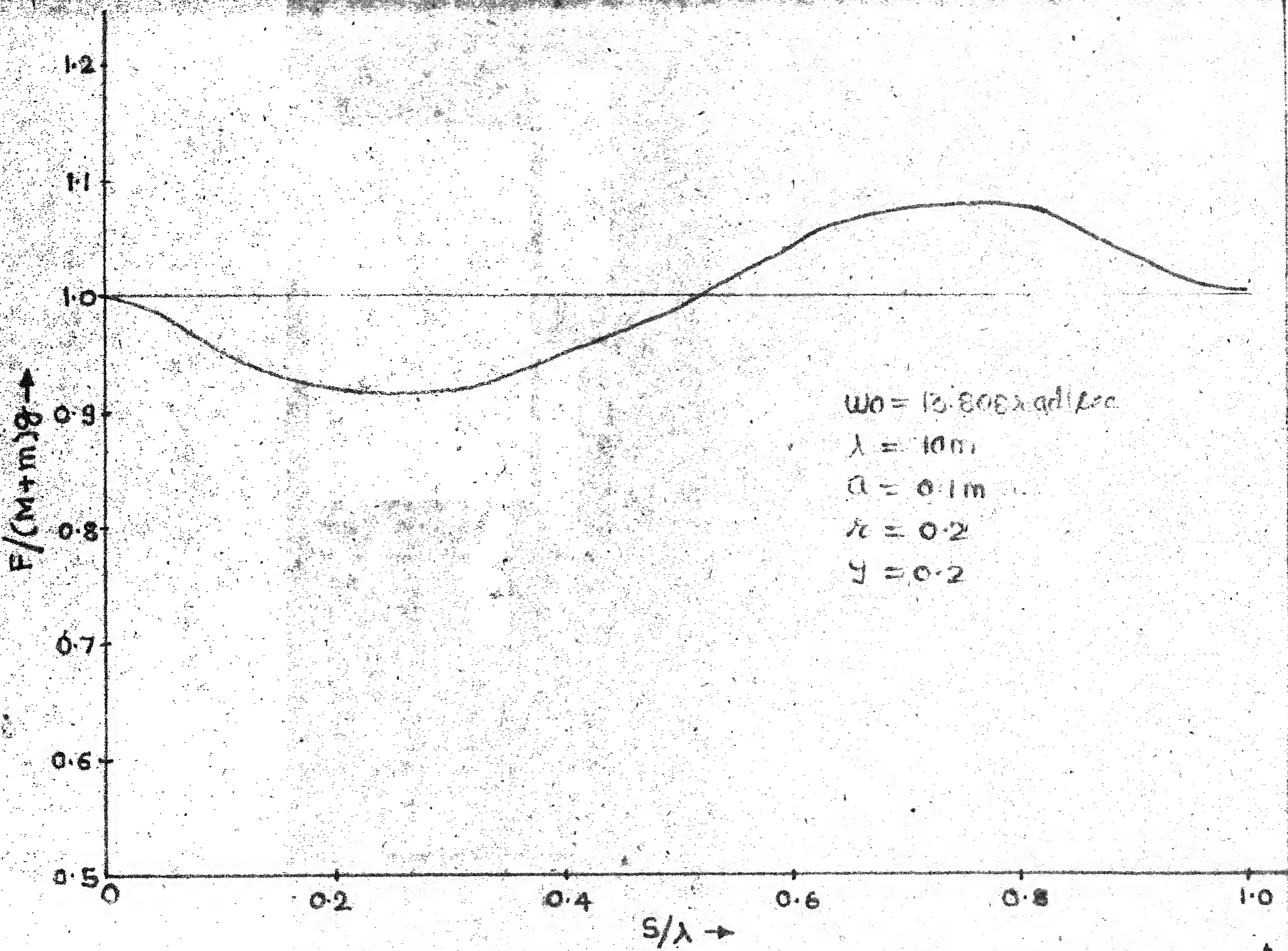
FIG. 3·4


$\omega_0 = 13.808$ rad/sec
$\lambda = 10$m
$a = 0.1$m
$r = 0.2$
$y = 0.2$
Z/a
S/λ
1.1
0.9
0.7
0.5
0.3
0.1
0
-0.1
-0.3
-0.5
-0.7
-0.9
-1.1
0.2
0.4
0.6
0.8
1.0
FIG 3.5



·10
·08
·06
·04
·02
0
-·02
-·04
-·06
-·08
-·10
0·2
0·4
0·6
0·8
1·0
$\ddot{Z}/g$
S/λ
Wo = 13·808 rad/sec
λ = 10m
a = 0·1m
x = 0·2
y = 0·2

FIG. 3·6



1·2
1·1
1·0
0·9
0·8
0·7
0·6
0·5
F/(M+m)g →
0
0·2
0·4
0·6
0·8
1·0
S/λ →
ωo = 13·808 rad/sec
λ = 10m
a = 0·1m
r = 0·2
y = 0·2

FIG. 3-7

CHAPTER 4

RESPONSE OF VEHICLE TO PERIODICALLY OCCURRING LOW JOINTS

INTRODUCTION

A rail track consists of fishplated rail joint and is supported on sleeper and ballast. The sleepers hold the rail in correct alignment and transmit the track load to the ballast over a wide area. A rail joint is the weakest part of track and a study of the effect of the joints on vertical response and track forces is very important from the point of view of design of the vehicle and track. A common feature of these joints is the hogging of the rails with passage of heavily loaded vehicle. When hogging exceeds a certain limit, the joints are called low joints. The shape of the joint or pulse (as frequently used in subsequent chapters) can be assumed to be of the following form (fig.4.1)

$$Y(S) = -h\ (1+\sin\Omega\ (S-L) \quad \text{when } L-\frac{1}{2} \leq S \leq L$$

$$= -h(1-\sin\Omega(S-L) \quad \text{when } L \leq S \leq L+\frac{1}{2}$$

$$= 0 \text{ otherwise.}$$

where $\Omega = \frac{\pi}{1}$ and we have considered only one pulse.

In this chapter first, response parameters for single pulse are calculated by using convolution integral approach. In section 4.12, response for series of pulses of different 1 and h are obtained by considering any n_{th} pulse and then summing up responses of each pulse. In section 4.13, a special

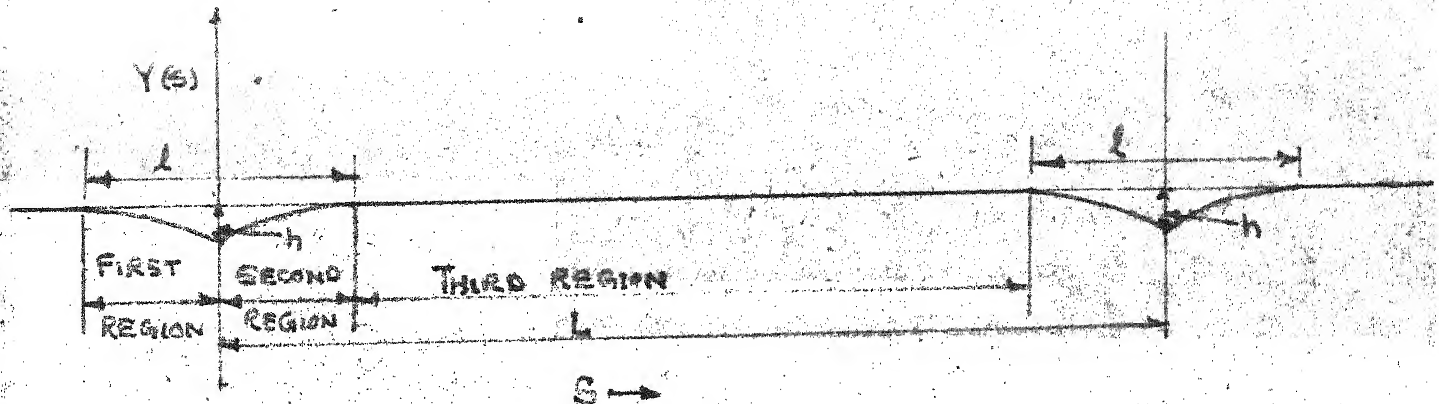


FIG. 4·1

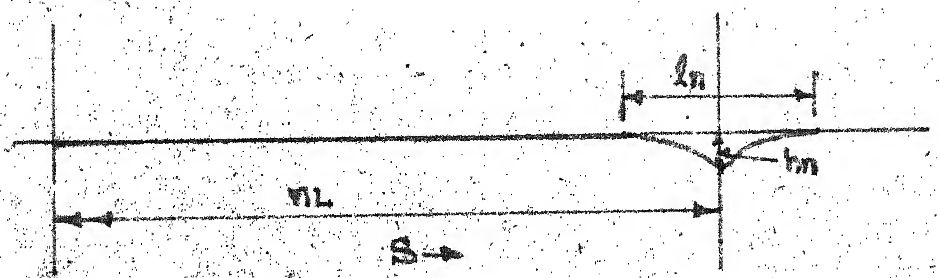


FIG. 4·2

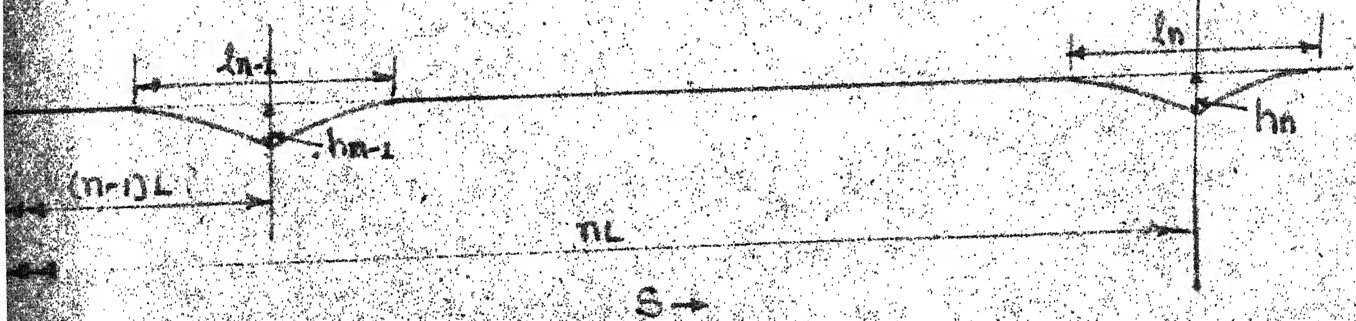


FIG. 4·3

case of same l and h is considered. Since these joints occur periodically, equivalent Fourier series can be considered and response statistics are obtained in section 4.21. Following parameters are calculated in each case.

i) Absolute displacement $Z(S)$ of sprung mass M.

ii) Absolute acceleration $\ddot{Z}(S)$ of sprung mass M.

iii) Contact Force $F(S)$.

4.1 Single Pulse Excitation

Equation of motion for single degree of freedom model in space coordinates is

$$v^2 Z'' + 2\zeta\omega_0 v Z' + \omega_0^2 Z = 2\zeta\omega_0 v Y' + \omega_0^2 Y \quad \text{from equation 2.3}$$

Solution of equation of motion:

Since $Y(S)$ is defined separately for three different region, we have to calculate response parameters separately for each region. Response for each region is calculated by using convolution integral approach.

(a) First Region ($L - \frac{1}{2} \leq S \leq L$)

Homogenous solution: This can be written directly from equation 2.7 as

$$x_{ih} = \beta_i e^{\frac{\alpha_i S}{v}}$$

where β_i and α_i are same as defined in equation 2.7.

Non-homogenous solution: From equation 2.9 we have

$$x_{inh} = \int_{L-\frac{1}{2}}^{S} \frac{\Phi_i}{M_i} h_i(S-p) \left\{ 2\zeta\omega_0 v \frac{dY}{dp} + \omega_0^2 Y \right\} dp$$

where $Y = -h(1 + \sin\Omega(p - L))$

$$\frac{dY}{dp} = -h\Omega\cos\Omega(p - L)$$

$$h_i(S-p) = \frac{e^{\frac{\alpha_i}{v}(S-p)}}{v} \qquad i = 1,2$$

substituting these values in above equation, we get

$$x_{inh} = -\int_{L-\frac{1}{2}}^{S} \frac{\left(he^{\frac{\alpha_i}{v}(S-p)}\right)}{v} \frac{\phi_i}{M_i}\left\{(2\zeta\omega_0 v\Omega\cos\Omega(p-L) + \omega_0^2 (1 + \sin\Omega(p - L))\right\} dp.$$

$$= \frac{-h\phi_i}{M_i v}\left\{-\frac{\omega_0^2}{u}\left(1 - e^{u(S-L+\frac{1}{2})}\right) + \frac{T}{A}\left(-u\sin(\Omega(S-L)+\Upsilon) - \Omega\cos(\Omega(S-L)+\Upsilon)\right) + \frac{T}{A}e^{u(S-L+\frac{1}{2})}\left(-u\cos\Upsilon + \Omega\sin\Upsilon\right)\right\} \qquad 4.1$$

where $\alpha_i = (-\zeta \pm \sqrt{\zeta^2 - 1})$

$$u = \frac{\alpha_i}{v}$$

$$r = \frac{\Pi v}{l\,\omega_0} = \frac{\Omega v}{\omega_0}$$

$$\Upsilon = \tan^{-1} 2\zeta r$$

$$A = \frac{(-\alpha_i)^2}{(v)^2} + \Omega^2$$

$$M_i = 2\phi_i^2(\zeta\omega_0 + \alpha_i) \qquad i = 1,2$$

and $T = \omega_0^2(1 + (2\zeta r)^2)^{\frac{1}{2}}$

hence

i) <u>Absolute displacement:</u> $Z(S)$

$$Z(S) = \sum_{i=1}^{2}\phi_i x_{ih} + \sum_{i=1}^{2}\phi_i x_{inh} \text{ from equation 2.10}$$

$$= \sum_{i=1}^{2} \phi_i \beta_i e^{\frac{\alpha_i S}{V}} + \sum_{i=1}^{2} \frac{h \phi_i^2}{M_i V} \left(\frac{\omega_0^2}{u} \left(1 - e^{u(S-L+\frac{1}{2})}\right) \right.$$

$$+ \frac{T}{A} \left\{ u \sin(\Omega(S-L) + \gamma) + \Omega \cos(\Omega(S-L) + \gamma) \right\}$$

$$\left. + \frac{T}{A} e^{u(S-L+\frac{1}{2})} \left(u \cos\gamma - \Omega \sin\gamma \right) \right) \qquad 4.2$$

ii) <u>Absolute Acceleration</u> $\ddot{Z}(S)$

$$\ddot{Z}(S) = \sum_{i=1}^{2} \phi_i \alpha_i^2 \beta_i e^{\frac{\alpha_i S}{V}} + \sum_{i=1}^{2} \frac{h \phi_i^2}{M_i} V \left(-\frac{\omega_0^2}{u} e^{u(S-L+\frac{1}{2})} \right.$$

$$+ \frac{T}{A} \Omega^2 \left(-u \sin(\Omega(S-L) + \gamma) - \Omega \cos(\Omega(S-L) + \gamma) \right)$$

$$\left. + \frac{T}{A} u^2 e^{u(S-L+\frac{1}{2})} \left(u \cos\gamma - \Omega \sin\gamma \right) \right) \qquad 4.3$$

iii) <u>Contact Force</u> (F(S)

$$F(S) = M\ddot{Z}(S) + m\ddot{Y}(S) + (M + m)g$$

substituting values of $\ddot{Y}(S)$ and $\ddot{Z}(S)$ we get

$$F(S) = M \left(\sum_{i=1}^{2} \phi_i \alpha_i^2 \beta_i e^{\frac{\alpha_i S}{V}} + \sum_{i=1}^{2} \frac{h \phi_i^2}{M_i} V \left(-\frac{\omega_0^2}{u} e^{u(S-L+\frac{1}{2})} \right. \right.$$

$$+ \frac{T}{A} \Omega^2 \left(-u \sin(\Omega(S-L) + \gamma) - \Omega \cos(\Omega(S-L) + \gamma) \right) + \frac{T}{A}$$

$$\left. \left. u^2 \cdot e^{u(S-L+\frac{1}{2})} \left(u \cos\gamma - \Omega \sin\gamma \right) \right) \right) + m \omega_0^2 r^2 h$$

$$\sin(\Omega(S-L)) + (M + m)g \qquad 4.4$$

(b) <u>Second Region:</u> ($L \leq S \leq L + \frac{1}{2}$)

Homogenous solution: This can be written directly from equation 3.7 as

$$x_{ih} = \beta_i \; e^{\frac{\alpha_i s}{v}}$$

Non-homogenous solution: From equation 2.8 we have

$$x_{inh} = \int_{L-\frac{1}{2}}^{L} h_i(S-p) \left\{ 2\zeta\omega_0 \frac{dY}{dp} v + \omega_0^2 Y \right\} \frac{\phi_i}{M_i} \, dp$$

$$+ \int_{L}^{S} h_i(S-p) \frac{\phi_i}{M_i} \left\{ 2\zeta\omega_0 \frac{dY}{dp} v + \omega_0^2 Y \right\} dp$$

where first integral has solved in equation 4.1

For second integral

$$Y = -h \; (1 - \sin \Omega (p - L) \;)$$

$$\frac{dY}{dp} = h\Omega \cos \; (\Omega \; (p - L))$$

and $h_i(S-p) = \dfrac{e^{\frac{\alpha_i}{v}(S-p)}}{V}$ $\quad i = 1,2$

$$x_{inh} = \int_{L-\frac{1}{2}}^{L} - he^{\frac{\alpha_i}{v}(S-P)} \frac{\phi_i}{M_i v} (2\zeta\omega_0 v\Omega \; \cos \Omega (P - L)$$

$$+ \; \omega_0^2 (1 + \sin \Omega (p-L)) dp \; + \int_{L}^{S} \frac{- h \; e^{\frac{\alpha_i}{V}(S-p)}}{M_i v} \; \phi_i$$

$$- \; 2\zeta\omega_0 v\Omega \cos \Omega (p - L) \; + \; \omega_0^2 (1 - \sin\Omega (p - L)) \, dp \qquad 4.5$$

$$= \frac{h \; \phi_i}{M_i v} \left\{ \frac{\omega_0^2}{u} \left(1 - e^{\frac{ul}{2}} \right) + \frac{T}{A} \left(\Omega \cos\gamma \; + u \sin\gamma \right) + \frac{T}{A} \; e^{\frac{ul}{2}} \right.$$

$$(u \cos\gamma - \Omega \sin\gamma) \; + \; \frac{\omega_0^2}{u} \left(1 - e^{u(S-L)} \right) - \frac{T}{A} \; \left\{ u \sin \right.$$

$$(\Omega (S-L) \; + \gamma) + \Omega \cos \; (\Omega (S-L) + \gamma \;) \Big\} + \frac{T}{A} \; e^{u(S-L)}$$

$$\left. \Big\{ u \sin\gamma + \Omega \cos\gamma \Big\} \right\} \qquad 4.6$$

hence

i) <u>Absolute displacement</u> : Z(S)

$$Z(S) = \sum_{i=1}^{2} x_i h\, \phi_i + \sum_{i=1}^{2} x_{inh}\, \phi_i$$

$$= \sum_{i=1}^{2} \beta_i\, \phi_i\, e^{\frac{\alpha_i S}{v}} + \sum_{i=1}^{2} \frac{h\, \phi_i^{2}}{M_i v} \left(\omega_0^2 \left(\frac{1}{u} \left(1 - e^{\frac{ul}{2}} \right)\right)\right.$$

$$+ \frac{T}{A} \left(\Omega \cos\gamma + u \sin\gamma \right) + \frac{T}{A}\, e^{\frac{ul}{2}} \left(u \cos\gamma - \Omega \sin\gamma \right)$$

$$+ \frac{\omega_0^2}{u} \left(1 - e^{u(S-L)}\right) - \frac{T}{A} \Big\{ u \sin(\Omega(S-L) + \gamma) + \Omega$$

$$\left.\cos(\Omega(S-L) + \gamma) \Big\} + \frac{T}{A}\, e^{u(S-L)} \left(u \sin\gamma + \Omega \cos\gamma \right) \right) \qquad 4.7$$

ii) <u>Absolute acceleration</u> : $\ddot{Z}(S)$

$$\ddot{Z}(S) = \sum_{i=1}^{2} \phi_i\, \alpha_i^{2}\, \beta_i\, e^{\frac{\alpha_i S}{v}} + \sum_{i=1}^{2} \frac{h\, \phi_i^{2}\, v}{M_i} \Big\{ -u\omega_0^2$$

$$e^{u(S-L)} - \frac{T}{A}\, \Omega^2 \Big(- u \sin(\Omega(S-L) + \gamma) - \Omega \cos(\Omega(S-L)$$

$$+ \gamma) \Big) + \frac{T}{A}\, u^2\, e^{u(S-L)} \left(u \sin\gamma + \Omega \cos\gamma \right) \Big\} \qquad 4.8$$

iii) <u>Contact Force:</u> F(S)

$$F(S) = M\, \ddot{Z}(S) + m\ddot{Y}(S) + (M + m) g$$

substituting values of $\ddot{Z}(S)$ and $\ddot{Y}(S)$, we get

$$F(S) = M \Bigg[\sum_{i=1}^{2} \phi_i\, \alpha_i^{2}\, \beta_i e^{\frac{\alpha_i S}{v}} + \sum_{i=1}^{2} \frac{h\, \phi_i^{2}\, v}{M_i} \Big\{ -u\, \omega_0^2$$

$$e^{u(S-L)} - \frac{T\Omega^2}{A} \Big(- u \sin\left(\Omega(S-L) + \gamma \right) - \Omega \cos\left(\Omega(S-L) + \gamma \right)$$

$$\Big) + \frac{T}{A}\, u^2\, e^{u(S-L)} \left(u \sin\gamma + \Omega \cos\gamma \right) \Big\} \Bigg]$$

$$- m\, \omega_0^2 r^2\, h \sin \Omega (S-L) + (M + m) g \qquad 4.9$$

(c) Third Region: $(S \geq L + \frac{1}{2})$

In thisregion, there is no ground induced excitation since Y(S) is zero. Hence this part contains only transient vibration which dies out as distance increases. The response for this part can be written in the form.

$$Z(S) = Z_{So}\, g(S - S_o) + \dot{Z}_{So}\, h(S - S_o)$$

hence,

i) Absolute displacement: Z(S)

$$Z(S) = Z_{So}\, g(S - S_o) + \dot{Z}_{So}\, h(S - S_o) \qquad 4.10$$

where $S_o = L + \frac{1}{2}$

$$Z_{So} = Z(S_o) = \sum_{i=1}^{2} \frac{h\,\Phi_i^2}{M_i V} \left(\frac{\omega_0^2}{u} \left(1 - e^{\frac{ul}{2}} \right) + \frac{T}{A} \left(\Omega \cos\gamma + u \sin\gamma \right) + \frac{T}{A}\, e^{\frac{ul}{2}} \left(u \cos\gamma - \Omega \sin\gamma \right) + \frac{\omega_0^2}{u} \right.$$

$$\left. (1 - e^{\frac{ul}{2}}) + \frac{T}{A} \left\{ u \cos\gamma - \Omega \sin\gamma \right\} - \frac{T}{A}\, e^{\frac{ul}{2}} \left\{ u \sin\gamma + \Omega \cos\gamma \right\} \right)$$

$$+ \sum_{i=1}^{2} \Phi_i\, \beta_i\, e^{\frac{\alpha_i}{V}(L + \frac{1}{2})} \qquad 4.11$$

$$\dot{Z}_{So} = \dot{Z}(S_o) = \sum_{i=1}^{2} \Phi_i\, \alpha_i\, \beta_i\, e^{\frac{\alpha_i}{V}(L + \frac{1}{2})} + \sum_{i=1}^{2} \frac{h\,\Phi_i^2}{M_i}$$

$$\left(-\Omega \frac{T}{A} \left\{ -u \sin\gamma - \Omega \cos\gamma \right\} - \omega_0^2 e^{\frac{ul}{2}} + \frac{T}{A}\, e^{\frac{ul}{2}} \right.$$

$$\left. \left(u \sin\gamma + \Omega \cos\gamma \right) \right) \qquad 4.12$$

$$g(S-S_o) = e^{\frac{-\zeta\omega_0}{v}(S-S_o)} \left\{ \cos\frac{\omega_d}{v}(S-S_o) + \frac{\zeta}{(1-\zeta^2)^{\frac{1}{2}}} \right.$$

$$\left. \sin\frac{\omega_d}{v}(S-S_o) \right\} \qquad 4.13$$

$$h(S-S_o) = \frac{e^{\frac{-\zeta\omega_0}{v}(S-S_o)}}{\omega_d} \sin\frac{\omega_d}{v}(S-S_o) \qquad 4.14$$

$$\omega_d = \omega_0(1-\zeta^2)^{\frac{1}{2}} \qquad 4.15$$

ii) <u>Absolute acceleration:</u> $\ddot{Z}(S)$

$$\ddot{Z}(S) = Z_{So}\,\ddot{g}(S-S_o) + \dot{Z}_{So}\,\ddot{h}(S-S_o) \qquad 4.16$$

where $\ddot{g}(S-S_o) = e^{\frac{-\zeta\omega_0}{v}(S-S_o)} \left(\sin\frac{\omega_d}{v}(S-S_o) \right.$

$$\left\{ \frac{\zeta}{(1-\zeta^2)^{\frac{1}{2}}} (\zeta^2\omega_0^2 - \omega_d^2) + 2\zeta\omega_0\omega_d \right\} + \cos\frac{\omega_d}{v}$$

$$\left. (S-S_o) \left(\frac{\zeta}{(1-\zeta^2)^{\frac{1}{2}}} (-2\zeta\omega_0\omega_d) + \zeta^2\omega_0^2 - \omega_d^2 \right) \right)$$

$$\ddot{h}(S-S_o) = \frac{e^{-\frac{\zeta\omega_0}{v}(S-S_o)}}{\omega_d} \left((\zeta^2\omega_0^2 - \omega_d^2) \sin\frac{\omega_d}{v} \right.$$

$$\left. (S-S_o) - 2\zeta\omega_0\omega_d \cos\frac{\omega_d}{v}(S-S_o) \right)$$

iii) <u>Contact Force:</u> $F(S)$

$$F(S) = M\ddot{Z}(S) + (M+m)g$$

$$= M(Z_{So}\ddot{g}(S-S_o) + \dot{Z}_{So}\ddot{h}(S-S_o)) + (M+m)g \qquad 4.17$$

4.1.2. <u>Excitation due to Series of Pulses</u>

In the previous analysis of single pulse excitation, we have calculated response parameters by considering single pulse excitation only. When series of pulses of the some shape best of different size (different l and h)

occur periodically, then response is calculated by considering any n_{th} pulse and then summing up response due to previous pulses (By using superposition principle).

For any n_{th} pulse, shape can be described as (Fig. 4.2)

$$Y_n(S) = -h_n(1 + \sin \Omega n(S - nL)) \text{ when } nL - \frac{l_n}{2} \leq S \leq nL$$

$$= - h_n(1 - \sin \Omega n(S - nL)) \text{ when } nL \leq S \leq nL + \frac{l_n}{2}$$

$= 0$ otherwise.

where $\Omega n = \pi / l_n$

$n = 1, 2 - - -$

In Fig. 4.3, l_{n-1} and l_n are length of $(n-1)_{th}$ pulse and n_{th} pulses respectively. h_{n-1} and h_n are depths of $(n-1)_{th}$ and n_{th} pulse respectively.

Response parameters for any n_{th} pulse can be written directly from the previous single pulse excitation analysis. Hence for three region, response paramter will be as follows:

(a) <u>First Region:</u> $nL - \frac{l_n}{2} \leq s \leq nL$

i) <u>Absolute displacement:</u> $Z_{n,1}(S)$

$$Z_{n,1}(S) = \sum_{i=1}^{2} \phi_i \beta_i e^{\frac{\alpha_i}{v}\left(S - nL + \frac{l_n}{2}\right)} + \sum_{i=1}^{2} \frac{h_n \phi_i}{M_i v} \left(\frac{\omega_o^2}{u} \left(1 - e^{u\left(S - nL + \frac{l_n}{2}\right)}\right) + \frac{T_n}{A_n} \left\{ u \sin(\Omega n(S - nL) + \Upsilon n) + \Omega n \cos(\Omega n(S - nL) + \Upsilon n) \right\} + \frac{T_n}{A_n} e^{u\left(S - nL + \frac{l_n}{2}\right)} \left\{ u \cos \Upsilon n - \Omega n \sin \Upsilon n \right\} \right) \quad 4.18$$

where $Z_{n,1}(S)$ is the absolute displacement due to n_{th} pulse only.

$$T_n = \left[(2\zeta \omega_0^2 r_n)^2 + \omega_0^4 \right]^{\frac{1}{2}}$$

$$= \omega_0^2 \left[1 + (2\zeta r_n)^2 \right]^{\frac{1}{2}}$$

$$r_n = \pi v / l_n \omega_0$$

$$A_n = \left[\left(-\frac{\alpha_i}{v} \right)^2 + (\Omega_n)^2 \right]$$

$$\gamma_n = \tan^{-1} 2\zeta r_n$$

$$u = \alpha_i / v$$

ii) <u>Absolute acceleration:</u> $\ddot{Z}_{n,1}(S)$

$$\ddot{Z}_{n,1}(S) = \sum_{i=1}^{2} \beta_i \alpha_i^2 \Phi_i \, e^{\frac{\alpha_i}{v}\left(S - nL + \frac{l_n}{2}\right)} + \sum_{i=1}^{2}$$

$$\frac{h_n \Phi_i^2 v}{M_i} \left(- u \omega_0^2 . \; e^{u\left(S-nL + \frac{l_n}{2}\right)} + \frac{T_n}{A_n} \Omega_n^2 \right[$$

$$- u \sin \left(\Omega_n (S - nL) + \gamma_n \right) - \Omega_n \cos \left(\Omega_n (S-nL) + \gamma_n \right] $$

$$+ \frac{T_n}{A_n} u^2 e^{u\left(S-nL + \frac{l_n}{2}\right)} \left[u \cos \gamma_n - \Omega_n \sin \gamma_n \right] \Big) \qquad 4.19$$

where $\ddot{Z}_{n,1}(S)$ is the acceleration due to n_{th} pulse only.

iii) <u>Contact Force:</u> $F_{n,1}(S)$

$$F_{n,1}(S) = M \ddot{Z}_{n,1}(S) + m \ddot{Y}_n(S) + (M + m) g$$

$$= M \Big(\sum_{i=1}^{2} \beta_i \alpha_i^2 \Phi_i e^{\frac{\alpha_i}{v}\left(S - nL + \frac{l_n}{2}\right)} +$$

$$\sum_{i=1}^{2} \frac{h_n \Phi_i^2 v}{M} \Big(-u \omega_0^2 e^{u\left(S - nL + \frac{l_n}{2}\right)} + \frac{T_n}{A_n} \Omega_n^2$$

$$\Big\}- u\sin(\Omega n(S-nL)+\gamma n)-\Omega n\cos(\Omega n(S-nL)+\gamma n$$

$$\Big\}+\frac{T_n}{2A_n}\,u^2\,e^{u(S-nL+\frac{l_n}{2})}\Big\}u\cos\gamma n-\Omega n\sin\gamma n\Big\}+\frac{T_n}{A_n}\frac{u^2}{2}$$

$$e^{u(S-nL+\frac{l_n}{2})}\Big\}u\cos\gamma n-\Omega n\sin\gamma n\Big\}\Big)+m\,\omega_0^2$$

$$\gamma_n^2\,h_n\sin\Omega n\,(S-nL)+(M+m)\,g \qquad 4.20$$

(b) Second Region: $nL \leq S \leq nL + \frac{l_n}{2}$

(i) Absolute Displacement: $Z_{n,2}(S)$.

$$Z_{n,2}(S) = \sum_{i=1}^{2} \beta_i\,\phi_i\,e^{\frac{\alpha_i}{v}(S-nL)} + \sum_{i=1}^{2}\frac{h_n\,\phi_i^2}{M_i v}$$

$$\Big\{\frac{\omega_0^2}{u}\Big(1-e^{\frac{u l_n}{2}}\Big)+\frac{T_n}{A_n}\Big(\Omega_n\cos\gamma n+u\sin\gamma n\Big)+\frac{T_n}{A_n}$$

$$e^{\frac{u l_n}{2}}\,(u\cos\gamma n-\Omega_n\sin\gamma n)+\frac{\omega_0^2}{u}\,(1-e^{u(S-nL)})-\frac{T_n}{A_n}$$

$$\Big\}\,u\sin(\Omega n(S-nL)+\gamma n)+\Omega n\cos(\Omega n(S-nL)+\gamma n)+\frac{T_n}{A_n}$$

$$e^{u(S-nL)}\Big(u\sin\gamma n+\Omega n\cos\gamma n\Big)\Big\}\Big\} \qquad 4.21$$

where $Z_{n,2}(S)$ is the response due to the n_{th} pulse only.

ii) Absolute acceleration: $\ddot{Z}_{n,2}(S)$.

$$\ddot{Z}_{n,2}(S) = \sum_{i=1}^{2}\phi_i\,\alpha_i^2\,\beta_i\,e^{\frac{\alpha_i}{v}(S-nL)} + \sum_{i=1}^{2}\frac{h_n\,\phi_i^2}{M_i}\,v$$

$$\Big(-u\,\omega_0^2 e^{u(S-nL)}-\frac{T_n}{A_n}\Omega_n^2\Big(u\sin(\Omega n(s-nL)+\gamma n)-\Omega n\cos$$

$$(\Omega n(S-nL)+\gamma n\Big)\Big)+\frac{T_n}{A_n}\,u^2\,e^{u(S-nL)}\Big\}\,u\sin\Omega n+\Omega n\cos\gamma n)$$

4.22

Where

$\ddot{Z}_{n,2}(S)$ is the acceleration due to the n_{th} pulse only.

iii) Contact Force: $F_{n,2}(S)$

$$F_{n,2}(S) = M\ddot{Z}_{n,2}(S) + m\ddot{y}_n(S) + (M+m)g$$

$$= M\Big(\sum_{i=1}^{2} \phi_i \alpha_i^2 \beta_i e^{\frac{\alpha_i}{v}(S-nL)} + \sum_{i=1}^{2} \frac{h_n \phi_i^2}{M_i} v$$

$$\Big\{ -u\omega_o^2 e^{u(S-nL)} - \frac{T_n}{A_n}\Omega_n^2 \Big\{ -u\sin(\Omega n(S-nL)+\gamma n)$$

$$-\Omega n\cos(\Omega n(S-nL)+\gamma n)\Big\} + \frac{T_n}{A_n} u^2 e^{u(S-nL)}$$

$$\big(u\sin\gamma n + \Omega n\cos\gamma n\big)\Big\}\Big) - m\,\omega_o^2 r_n^2\, h_n \sin\Omega n(S-nL) + (M+m)g \quad (4.23)$$

Where $F_{n,2}(S)$ is the contact force due to the n_{th} pulse only.

Third Region: $S \geq nL + \frac{l_n}{2}$

(i) Absolute displacement: $Z_{n,3}(S)$

$$Z_{n,3}(S) = Z_{Sn}\, g(S-Sn) + \dot{Z}_{Sn}\, h(S-Sn) \qquad 4.24$$

Where $S_n = nL + \frac{l_n}{2}$

$$Z_{Sn} = Z(S_n) = \sum_{i=1}^{2} \frac{h_n \phi_i^2}{M_i v} \Big\{ \frac{2\omega_o^2}{u} \Big(1 - e^{\frac{u l_n}{2}} \Big)$$

$$+ \frac{T_n}{A_n}\Big\{\Omega n\cos\gamma n + u\sin\gamma n\Big\} + \frac{T_n}{A_n}(u\cos\gamma n - \Omega n\sin\gamma n)\big(e^{\frac{u l_n}{2}} - 1\big)$$

$$- \frac{T_n}{A_n} e^{\frac{u l_n}{2}} \big(u\sin\gamma n + \Omega n\cos\gamma n\big)\Big\} + \sum_{i=1}^{2} \phi_i \beta_i e^{\frac{\alpha_i}{v}\left(\frac{l_n}{2}\right)} \qquad 4.25$$

$$\dot{Z}_{Sn} = \dot{Z}(S_n) = \sum_{i=1}^{2} \phi_i \alpha_i \beta_i e^{\left(\frac{\alpha_i}{v}\frac{l_n}{2}\right)} + \sum_{i=1}$$

$$\frac{h_n \Phi_i^2}{M_i} \left(\left(- \frac{\Omega n T_n}{A_n} \right) \left\{ -u \sin \gamma n - \Omega n \cos \gamma n \right\} - \omega_0^2 e^{\frac{u l_n}{2}} + \right.$$

$$\left. \frac{T_n}{A_n} e^{\frac{u l_n}{2}} \left(u \sin \gamma n + \Omega n \cos \gamma n \right) \right) \qquad 4.26$$

$$g(s - s_n) = e^{\frac{-\zeta \omega_0}{v}(s-s_n)} \left\{ \cos \frac{\omega_d}{v} (s-s_n) + \frac{\zeta}{(1-\zeta^2)^{\frac{1}{2}}} \right.$$

$$\left. \sin \frac{\omega_d}{v} (s - s_n) \right\} \qquad 4.27$$

$$\text{or } g(s-s_n) = e^{-\frac{\zeta \omega_0}{v}(s-s_n)}$$

$$\left(\cos \frac{\omega_d}{v} (s-s_n) + \frac{\zeta}{(1-\zeta^2)^{\frac{1}{2}}} \sin \frac{\omega_d}{v} (s-sn) \right)$$

$$\text{and } h(s-s_n) = \frac{e^{-\frac{\zeta \omega_0}{v}(s-s_n)}}{\omega_d} \sin \frac{\omega_d}{v} (s-s_n) \qquad 4.28$$

$$\omega d = \omega_0 (1 - \zeta^2)^{\frac{1}{2}}$$

Where $Z_{n,3(s)}$ is the response due to n_{th} pulse only.

(ii) Absolute Acceleration: $\ddot{Z}_{n,3(s)}$

$$\ddot{Z}_{n,3(s)} = Z_{sn} \ddot{g}(s-s_n) + \dot{Z}_{sn} \ddot{h}(s-s_n) \qquad 4.29$$

where

$$\ddot{g}(s-s_n) = e^{-\frac{\zeta \omega_0}{v}(s-s_n)} \left(\sin \frac{\omega_d}{v} (s-s_n) \left\{ \frac{\zeta}{(1-\zeta^2)^{\frac{1}{2}}} \right. \right.$$

$$\left. (\zeta^2 \omega_0^2 - \omega d^2) + 2\zeta\omega_0\omega d \right\} + \cos \frac{\omega_d}{v} (s-s_n) \left(\frac{\zeta}{(1-\zeta^2)^{\frac{1}{2}}} \right.$$

$$\left. \left. (-2\zeta\omega_0\omega d) + \zeta^2 \omega_0^2 - \omega d^2 \right) \right) \qquad 4.30$$

$$\text{and } \ddot{h}(s-s_n) = \frac{e^{-\frac{\zeta \omega_0}{v}(s-s_n)}}{\omega d} \left(\sin \frac{\omega_d}{v} (s-s_n) \left\{ \zeta^2 \omega_0^2 \right. \right.$$

$$\left. \left. - \omega_d^2 \right\} - 2\zeta\omega_0\omega d \cos \frac{\omega_d}{v} (s - s_n) \right) \qquad 4.31$$

where $\ddot{Z}_{n,3}$ is the acceleration due to n_{th} pulse only.

(iii) <u>Contact force:</u> $F_{n,3}(S)$

$$F_{n,3}(S) = M\ddot{Z}_{n,3}(S) + (M+m)g$$

$$= M(\ddot{Z}_{Sn}g(S-S_n) + \dot{\ddot{Z}}_{Sn}h\ (S-S_n)) + (M+m)g \qquad 4.32$$

Here, above, we have calculated response parameters due due to the n_{th} pulse only. Response parameters for series of pulses are obtained by summing response parameters due to each pulse (using superposition principle).

1) <u>Absolute displacement:</u> $Z(S)$

$$Z(S) = \sum_{n=1}^{N} Z_{n,3}(S) + Z_{N+1,1}(S); \text{ when } (N+1)L - \frac{l_{N+1}}{2} \leq S \leq (N+1)L$$

$$= \sum_{n=1}^{N} Z_{n,3}(S) + Z_{N+1,2}(S); \text{ when } (N+1)L \leq S \leq (N+1)L + \frac{l_{N+1}}{2}$$

$$= \sum_{n=1}^{N+1} Z_{n,3}(S) \text{ when } S \geq (N+1)L + \frac{l_{N+1}}{2}$$

where N is the number of pulses passed. 4.33

2) <u>Absolute acceleration:</u> $\ddot{Z}(S)$

$$\ddot{Z}(S) = \sum_{n=1}^{N} \ddot{Z}_{n,3}(S) + \ddot{Z}_{N+1,1}(S); \text{ when } (N+1)L - \frac{l_{N+1}}{2} \leq S \leq (N+1)L$$

$$= \sum_{n=1}^{N} \ddot{Z}_{n,3}(S) + \ddot{Z}_{N+1,2}(S); \text{ when } (N+1)L \leq S \leq (N+1)L + \frac{l_{N+1}}{2}$$

$$= \sum_{n=1}^{N+1} \ddot{Z}_{n,3}(S) \text{ when } S \geq (N+1)L + \frac{l_{N+1}}{2} \qquad 4.34$$

3) <u>Contact Force:</u> F(S)

$$F(S) = M\ddot{Z}(S) + (M + m)g + m\ddot{Y}_n(S)$$

hence,

$$F(S) = M\left(\sum_{n=1}^{N}\ddot{Z}_{n,3}(S) + \ddot{Z}_{N+1,1}(S)\right) + (M+m)g + m\ddot{Y}_n(S)$$

when $(N+1)L - \frac{l_{N+1}}{2} \leq S \leq (N+1)L$

$$= M\left(\sum_{n=1}^{N}\ddot{Z}_{n,3}(S) + \ddot{Z}_{N+1,2}(S)\right) + (M+m)g + m\ddot{Y}_n(S)$$

When $(N+1)L \leq S \leq (N+1)L + \frac{l_{N+1}}{2}$

$$= M\left(\sum_{n=1}^{N+1}\ddot{Z}_{n,3}(S)\right) + (M+m)g$$

when $S \geq (N+1)L + \frac{l_{N+1}}{2}$ 4.35

4.13. Special Case

A special case can be considered when all the pulses are identical (same l and h). In that case pulse shape can be described as (Fig. 4.4).

$$Y_n(S) = -h(1+\sin\Omega(S-nL)); \text{ when } nL - \frac{l}{2} \leq S \leq nL$$

$$= -h(1-\sin\Omega(S-nL)); \text{ when } nL \leq S \leq nL + \frac{l}{2}$$

$$= 0 \text{ otherwise,}$$

where $\Omega = \frac{\pi}{l}$ and $n = 1, 2 \cdots$

Response parameters for different region can be written directly from the previous section 4.12.

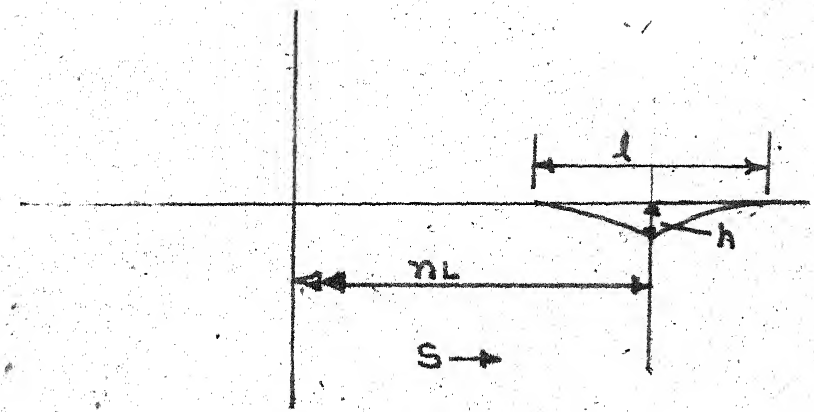


FIG 4.4

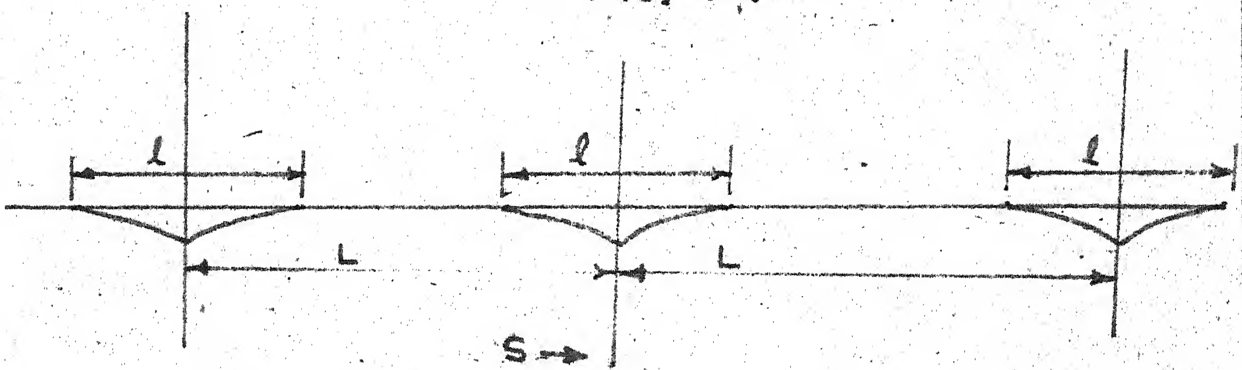


FIG. 4·5

(a) First Region : $nL - \frac{1}{2} \leq S \leq nL$

(i) Absolute Displacement: $Z_{n,1}(S)$

$$Z_{n,1}(S) = \sum_{i=1}^{2} \frac{h\,\Phi_i^2}{M_i V} \left(\frac{\omega_o^2}{u}\left(1 - e^{u(S-nL+\frac{1}{2})}\right) + \frac{T}{A}\left\{ u \sin\left(\Omega(S-nL)+\gamma\right) + \Omega\cos\left(\Omega(S-nL)+\gamma\right)\right\} + \frac{T}{A} e^{u(S-nL+\frac{1}{2})} \left\{ u\cos\gamma - \Omega\sin\gamma \right\}\right) + \sum_{i=1}^{2} \Phi_i\, \beta_i\, e^{\frac{\alpha_i}{V}(S-nL+\frac{1}{2})} \qquad 4.36$$

ii) Absolute acceleration: $\ddot{Z}_{n,1}(S)$

$$\ddot{Z}_{n,1}(S) = \sum_{i=1}^{2} \beta_i\, \alpha_i^2\, \Phi_i\, e^{\frac{\alpha_i}{V}(S-nL+\frac{1}{2})} + \sum_{i=1}^{2} \frac{h\,\Phi_i^2}{M_i} V\left(-u\omega_o^2\, e^{u(S-nL+\frac{1}{2})} + \frac{T}{A}\Omega^2 \left(-u\sin\left(\Omega(S-nL)+\gamma\right) - \Omega\cos\left(\Omega(S-nL)+\gamma\right)\right) + \frac{T}{A} u^2 e^{u(S-nL+\frac{1}{2})}\left(u\cos\gamma - \Omega\sin\gamma\right)\right) \qquad 4.37$$

iii) Contact Force: $F_{n,1}(S)$

$$F_{n,1}(S) = M\,\ddot{Z}_{n,1}(S) + m\,\ddot{Y}_n(S) + (M+m)g$$

$$= M \left\{ \sum_{i=1}^{2} \beta_i\, \alpha_i^2\, \Phi_i\, e^{\frac{\alpha_i}{V}(S-nL+\frac{1}{2})} + \sum_{i=1}^{2} \frac{h\,\Phi_i^2}{M_i} V\left(\right.\right.$$

$$\left(-u\,\omega_o^2 e^{u(S-nL+\frac{1}{2})} + \frac{T}{A}\,\Omega^2 \left(-u\sin\left(\Omega(S-nL)+\gamma\right) - \Omega\cos\left(\Omega(S-nL)+\gamma\right)\right) + \frac{T}{A} u^2 e^{u(S-nL+\frac{1}{2})} \left(u\cos\gamma - \Omega\sin\gamma \right)\right) + m\,\omega_o^2 r^2 h \sin\Omega(S-nL) + (M+m)g \qquad 4.38$$

(b) Second Region: $nL - \frac{1}{2} \leq S \leq nL$

i) Absolute Displacement: $Z_{n,2}(S)$

$$Z_{n,2}(S) = \sum_{i=1}^{2} \beta_i \phi_i e^{\frac{\alpha_i}{v}(S-nL)} + \sum_{i=1}^{2} \frac{h\,\phi_i^2}{M_i v} \left(\frac{\omega_o^2}{u} \left(1 - e^{\frac{ul}{2}} \right) + \frac{T}{A}(\Omega\cos\gamma + u\sin\gamma) + \frac{T}{A} e^{\frac{ul}{2}} \left(u\cos\gamma - \Omega\sin\gamma \right) + \frac{\omega_o^2}{u}\left(1 - e^{u(S-nL)}\right) - \frac{T}{A}\left(u\sin\left(\Omega(S-nL)+\gamma\right) + \Omega\cos\left(\Omega(S-nL)+\gamma\right)\right) + \frac{T}{A} e^{u(S-nL)} (u\sin\gamma + \Omega\cos\gamma)\right) \qquad 4.39$$

ii) Absolute Acceleration: $\ddot{Z}_{n,2}(S)$

$$\ddot{Z}_{n,2}(S) = \sum_{i=1}^{2} \phi_i \alpha_i^2 \beta_i e^{\frac{\alpha_i}{v}(S-nL)} + \sum_{i=1}^{2} \frac{h\,\phi_i^2}{M_i} v \left(-u\,\omega_o^2 e^{u(S-nL)} - \frac{T}{A}\Omega^2 \left(-u\sin\left(\Omega(S-nL)+\gamma\right) - \Omega\cos\left(\Omega(S-nL)+\gamma\right)\right) + \frac{T}{A} u^2 e^{u(S-nL)} \left(u\sin\gamma + \Omega\cos\gamma \right)\right) \qquad 4.40$$

iii) Contact Force: $F_{n,2}(S)$

$$F_{n,2}(S) = M\,\ddot{Z}_{n,2}(S) + m\,\ddot{Y}_n + (M+m)g$$

$$= M\left(\sum_{i=1}^{2} \phi_i \alpha_i^2 \beta_i e^{\frac{\alpha_i}{v}(S-nL)} + \sum_{i=1}^{2} \frac{h\,\phi_i^2}{M_i} v \left(\right.\right.$$

$$\left\{ -u\omega_0^2\, e^{u(S-nL)} - \frac{T}{A}\Omega^2(-u \sin\,(\Omega(S-nL)+\gamma) - \Omega\cos\,(\Omega(S-nL)+\gamma) + \frac{T}{A}u^2\, e^{u(S-nL)} \left\{ u\sin\gamma + \Omega\cos\gamma \right\} \right\})$$

$$- m\,\omega_0^2 r^2\, h \quad \sin\Omega(S-nL) + (M+m)g \tag{4.41}$$

(C) <u>Third Region:</u> $S \geqslant nL + \frac{l}{2}$

i) Absolute displacement: $Z_{n,3}(S)$

$$Z_{n,3}(S) = Z_{Sn}\, g(S-Sn) + \dot{Z}_{Sn}\, h(S-Sn) \tag{4.42}$$

Where $S_n = nL + \frac{l}{2}$

$$Z_{Sn} = Z(S_n) = \sum_{i=1}^{2} \Phi_i\, \beta_i e^{\left(\frac{\alpha_i}{v}\, \frac{l}{2}\right)} + \sum_{i=1}^{2} \frac{h\,\Phi_i^2}{M_i V}$$

$$\left\{ \frac{\omega_0^2}{u} \left(1 - e^{\frac{ul}{2}} + \frac{T}{A}(\Omega\cos\gamma + u\sin\gamma) \right) + \frac{T}{A}\, e^{\frac{ul}{2}} \right.$$

$$(u\cos\gamma - \Omega\sin\gamma) + \frac{\omega_0^2}{2}$$

$$\left\{ 1 - e^{\frac{ul}{2}} \right\} - \frac{T}{A}\left\{ u\cos\gamma - \Omega\sin\gamma \right\} - \frac{T}{A}\, \frac{ul}{2}$$

$$\left.(u\sin\gamma + \Omega\cos\gamma)\right\} \tag{4.43}$$

$$\dot{Z}_{Sn} = \dot{Z}(Sn) = \sum_{i=1}^{2} \Phi_i\, \alpha_i\, \beta_i e^{\frac{\alpha_i l}{2v}} + \sum_{i=1}^{2} \frac{h\,\Phi_i^2}{M_i}$$

$$\left(-\Omega\frac{T}{A} \left\{ -u\sin\gamma - \Omega\cos\gamma \right\} - \omega_0^2 \quad e^{\frac{ul}{2}} + \frac{T}{A}\, e^{\frac{ul}{2}} \right.$$

$$\left. (u\sin\gamma + \Omega\cos\gamma) \right) \tag{4.44}$$

$$g(S-S_n) = e^{-\frac{\zeta\omega_0}{v}(S-S_n)} \left(\cos\frac{\omega_d}{v}(S-S_n) + \frac{\zeta}{(1-\zeta^2)^{\frac{1}{2}}} \sin\frac{\omega_d}{v}(S-S_n) \right) \quad 4.45$$

$$h(S-S_n) = \frac{e^{-\frac{\zeta\omega_0}{v}(S-S_n)}}{\omega_d} \sin\frac{\omega_d}{v}(S-S_n) \quad 4.46$$

ii) Absolute acceleration: $\ddot{Z}_{n,3}(S)$

$$\ddot{Z}_{n,3}(S) = Z_{Sn}\ddot{g}(S-S_n) + \dot{Z}_{Sn}\ddot{h}(S-S_n) \quad 4.47$$

where
$$\ddot{g}(S-S_n) = e^{-\frac{\zeta\omega_0}{v}(S-S_n)} \left(\left(\frac{\zeta}{(1-\zeta^2)^{\frac{1}{2}}} (\zeta^2\omega_0^2 - \omega d^2) + 2\zeta\omega_0\omega d \right) \sin\frac{\omega d}{v}(S-S_n) + \left(\frac{\zeta}{(1-\zeta^2)^{\frac{1}{2}}} (-2\zeta\omega_0\omega d) + \zeta^2\omega_0^2 - \omega d^2 \right) \cos\frac{\omega d}{v}(S-S_n) \right) \quad 4.48$$

$$\ddot{h}(S-S_n) = \frac{e^{-\frac{\zeta\omega_0}{v}(S-S_n)}}{d} \left(\sin\frac{\omega d}{v}(S-S_n)(\zeta^2\omega_0^2 - \omega d^2) - 2\zeta\omega_0\omega d \cos\frac{\omega d}{v}(S-S_n) \right) \quad 4.49$$

(iii) Contact Force: $F_{n,3}(S)$

$$F_{n,3}(S) = M\ddot{Z}_{n,3} + (M+m)g$$

$$= M(Z_{Sn}\ddot{g}(S-S_n) + \dot{Z}_{Sn}\ddot{h}(S-S_n)) + (M+m)g \quad 4.50$$

These are the response parameters in different sections due to n_{th} pulse only. Response parameters due to series of pulses are obtained in similar to previous section.

1) Absolute displacement: Z(S)

$$Z(S) = \sum_{n=1}^{N} Z_{n,3}(S) + Z_{N+1,1}(S); \text{ when } (N+1)L - \frac{1}{2} \leq S \leq (N+1)L$$

$$= \sum_{n=1}^{N} Z_{n,3}(S) + (Z_{N+1,2}(S)); \text{when } (N+1)L \leq S \leq (N+1)L + \frac{1}{2}$$

$$= \sum_{n=1}^{N+1} Z_{n,3}(S) \text{ when } S \geq (N+1)L + \frac{1}{2} \qquad 4.51$$

2) Absolute Acceleration: $\ddot{Z}(S)$

$$\ddot{Z}(S) = \sum_{n=1}^{N} \ddot{Z}_{n,3}(S) + \ddot{Z}_{N+1,1}(S), \text{ when } (N+1)L - \frac{1}{2} \leq S \leq (N+1)L$$

$$= \sum_{n=1}^{N} \ddot{Z}_{n,3}(S) + \ddot{Z}_{N+1,2}(S); \text{ when } (N+1)L \leq S \leq (N+1)L + \frac{1}{2} \qquad 4.52$$

$$= \sum_{n=1}^{N+1} \ddot{Z}_{n,3}(S) \qquad S \geq (N+1)L + \frac{1}{2}$$

3) Contact force: F(S)

$$F(S) = M\ddot{Z}(S) + (M+m)g + m\ddot{y}_n(S)$$

$$= M\left(\sum_{n=1}^{N} \ddot{Z}_{n,3}(S) + \ddot{Z}_{n+1}(S) \right) + (M+m)g + m\ddot{Y}_n(S); (N+1)L - \frac{1}{2} \leq S \leq (N+1)L$$

$$= M\left(\sum_{n=1}^{N} \ddot{Z}_{n,3}(S) + \ddot{Z}_{N+1,2}(S) \right) + (M+m)g$$

$$+ m\ddot{y}_n(S) \text{ when } (N+1)L \leq S \leq (N+1)L + \frac{1}{2}$$

$$= M\left(\sum_{n=1}^{N+1} \ddot{Z}_{n,3}(S) \right) + (M+m)g; \; S \geq (N+1)L + \frac{1}{2} \qquad 4.53$$

4.14 . Results

The data used for the calculation is for same bogie CR-28251 given table 3.1. In addition to that data, other data used is given in Table 4.1. (Page 55).

Following observations may be made from the results.

1. The dimensionless ratio r (= $\pi v/l\omega_0$) is an important parameter for the analysis, as it relates velocity, natural frequency and length of the pulse. For any fixed value of v and ω_0 decrease in r implies increase in pulse length and vice versa.

2. Response histories have been plotted as a function of distance alongthe track in figs. 4.6 to 4.8 for r = 1.1. Ratios of absolute displacement to the depth of pulse (h), absolute acceleration to acceleration due to gravity and contact force to the total weight of the vehicle have been plotted against S/l. Fixed parameters are ω_0=13.808 rad/sec. l = 1.5m, h = 5mm, and L = 13m.

There are two regions. First region is over the joint (forced vibration), and second region is beyond the joint until next pulse starts (Free vibration). Particular interest lies at the starting point of the next joint.

Maximum value of absolute displacement occurs near the end of joint while the maximum values of acceleration and contact force occur at the centre of pulse. After the

pulse, all parameters decay exponentially. For a joint spacing of 13 meters, response parameters, at the starting point of the next pulse is several order of magnitude less than the maximum value of response parameters due to preceding pulse. The interaction effect (response values at the starting point of successive pulses) is quite small.

3. Ratios of maximum value of absolute displacement to the depth h of the pulse, maximum value of accelerations to acceleration due to gravity (g), and maximum and minimum value of the contact force to the total weight of the vehicle have been plotted against r in figures 4.9 to 4.12. Fixed parameters are ω_0= 13.808 rad/sec. h = 5mm. Range of r is 0 to 4.4 (corresponding velocity range is 0 to 110 km/hr for 1.5 m pulse length).

4. Maximum absolute displacement increases upto r = 1.1, then reduces monotonically as r is increased. Amplification at r = 1.1 is of the order of 5.4.

5. Maximum absolute acceleration increases upto r=1.1, reduces sharply upto r = 2.0, then increases slowly as r is increased. Peak level at r = 1.1 is of the order of 0.51g.

6. Maximum contact force increases upto r = 1.1 reduces far r = 1.1 upto r = 1.8 and thereafter increases almost linearily, as r is increased. Peak value is 1.46 times the total weight of the vehicle.

7. Minimum contact force gives peak at $r = 1.1$, increases for $r > 1.1$ upto 2.0 and thereafter decreases almost linearly as r is increased. Peak value is 0.68 times the total weight of the vehicle.

8. Peak values of maximum absolute displacement, acceleration and contact force occurs for $r = 1.1$ (this corresponds to velocity of 27.5 km/hr for 1.5m pulse length).

9. The angle 2α $(=4h/l)$ is also an important parameter for the analysis, as it describes the shape of the pulse.

10. Maximum value of absolute displacement has been plotted against $2\alpha v$ for different values of r in fig. 4.14. Ratios of maximum value of acceleration to the g, and maximum and minimum value of the contact force to the total weight of the vehicle have been plotted against $2\alpha v$ for different values of r in figs. 4.14 to 4.16 Range of $2\alpha v$ is 0 to 2.0 (corresponding velocity range is 0 to 200 km/hr). Other fixed parameters are $2\alpha = 0.01334$ rad. $\omega_0 = 13.808$ rad/sec.

11. For a particular value of r and 2α, increase in veloci higher l as r is fixed, but since 2α is also fixed, increase in l increases h also.

12. All the responses parameters vary linearly with $2\alpha v$. Maximum values occur for r =1.1 Range of interest of $2\alpha v$ is generally 0.4 to 0.8. For $r = 1.1$, the maximum response values in this range are given in the following table.

TABLE 4.2

Response values	$2\alpha v = 0.4$	$2\alpha v = 0.8$
Z_{max}	30mm	58 mm
$\ddot{Z}_{max}/g$	0.55	1.10
$F_{max}/(M+m)g$	1.52	2.20
$F_{min}/(M+m)g$	0.65	0.29

13. $F_{max}/(M+m)g$ is called dynamic load factor (DLF) and it is seen from above that its value can be as high as 2.2.

14. In fig. 4.16, for r = 1.1, minimum value of contact force goes to zero at a velocity of about 80 km/hr. This velocity is important as vehicle losses its contact to the ground. For r = 1.2, limiting velocity is 105km/hr.

15. Figs. 4.17 to 4.19 have been plotted to determine the absolute displacement, acceleration and contact force of the vehicle due to excitation of one pulse, near the starting points of successive pulses. This interaction effect is important while determining response parameters due to series of pulses.

Z_{omax}, $\ddot{Z}_{omax}$ and F_{omax} are maximum values of displacement, acceleration and contact force on the pulse n =0, the excitation effect of which is to be considered near the starting point of successive pulses.

Z_{nmax}, $\ddot{Z}_{nmax}$ and F_{nmax} correspond to the maximum value of displacement, acceleration and contact force near the starting point of successive n pulses (n =1 to 5).

Displacement ratio (Z_{nmax}/Z_{omax}) acceleration ratio $(\ddot{Z}_{nmax}/\ddot{Z}_{omax})$ and contact force ratio

$\left(\frac{F_{nmax}}{(M+n)g} - 1\right) \Big| \left(\frac{F_{omax}}{(M+m)g} - 1\right)$ have been plotted in Figs. 4.17, 4.18 and 4.19 against n for different values of r on log scale. Fixed parameters are $\omega_0 = 13.808$ rad/sec. L = 13m. l = 1.5m. The reason of defining contact force ratio in the above form is that the value of $F_{nmax}/(M+m)g$ is nearly equal to 1. By substracting 1 from $F_{nmax}(M+m)g$ and dividing it by $(F_{omax}/(M+m)g) -1)$, it becomes easy and appropriate to draw it on log scale. A plot between the ratio of F_{nmax} to F_{omax} against n does not show any significant variation on log scale.

16. All the values decrease exponentially against n. Also it is seen that as r increases (velocity increases), all the ratios increase as clear from the following table.

TABLE 4·3

n	Velocity 30 km/hr			velocity 60 km/hr		
	Displacement ratio	Acce-Ratio	Contact force ratio	Displacement ratio	Acce. Ratio	Contact force Ratio
1	1.43×10^{-2}	8.7×10^{-3}	8.4×10^{-3}	7.2×10^{-2}	4.85×10^{-2}	4.15×10^{-2}
2	1.16×10^{-4}	7.9×10^{-5}	7.6×10^{-5}	5.3×10^{-3}	6.54×10^{-3}	5.6×10^{-3}
3	1.0×10^{-6}	1.21×10^{-5}	1.08×10^{-6}	4.3×10^{-4}	3.32×10^{-4}	7.98×10^{-4}
4	5.8×10^{-8}	6.56×10^{-7}	5.2×10^{-7}	3.12×10^{-5}	1.31×10^{-4}	1.13×10^{-4}
5	4.1×10^{-10}	1.10×10^{-10}	0.9×10^{-10}	1.08×10^{-6}	1.63×10^{-5}	1.42×10^{-5}

17. Displacement, acceleration and contact force ratios have been plotted against velocity for different values of $n = 1,5$ in figs. 4.20 to 4.22. Fixed parameters are $r = 1.1$, $\omega_0 = 13.808$ rad/sec. $L = 13$m. and $2\alpha = 0.01334$ rad. All the ratios increase with velocity. Also at low velicity, even at the first pulse (n=1), ratios are very small. A clear picture can be obtained from the numerical values given below.

Velocity (km/hr)	n	Displacement ratio	Acceleration ratio	Contact force ratio
30	1	0.0208	0.0233	0.0228
140	1	0.6280	0.585	0.575
30	5	4.1×10^{-10}	1×10^{-10}	0.9×10^{-10}
140	5	0.01127	0.01246	0.0120

It is seen that at low velocity (< 50km/hr), interaction effect is almost negligible at the 5th pulse.

18. Displacement and acceleration of the vehicle due to the excitation of 5 successive pulses are the summations of displacement and acceleration due to individual pulse, evaluated at that point. Contact force due to excitation of 5 pulses is obtained by following relation.

$$F_{nmax} = M \sum_{n=1}^{5} \ddot{Z}_{n,3}(S) + (M+m)g$$

Here also it is appropriate to substract $(M+m)g$ from F_{nmax} and divide it by $(M+m)g$.

$$Z_{nmax} = \sum_{n=1}^{5} \text{(Max values of } Z_{n,3(S)} \text{ evaluated near the}$$

starting point of 6_{th} pulse).

Z_{omax} = Max value of displacement on the pulse n =0.

$\ddot{Z}_{nmax} = \sum_{n=1}^{5}$ (Max value of $\ddot{Z}_{n,3(S)}$ evaluated near the starting point of 6_{th} pulse).

$\ddot{Z}_{omax}$ = Max value of acceleration on the pulse n =0.

F_{omax} = Max value of contact force on the pulse n =0.

Ratios of Z_{nmax} to Z_{omax}, $\ddot{Z}_{nmax}$ to $\ddot{Z}_{omax}$ and $(F_{nmax}/(M+m)g)-1)$ to $(F_{omax}/(M+m)g)-1)$ have been plotted against n for three velocities in figs. 4.23 to 4.25. Fixed parameters are r =1.1, ω_0= 13.808 rad/sec. L = 13m.

It is seen that-

i) For a velocity less than 50 km/hr, interaction effect of only previous pulse is significant.

ii) For a velocity lies between 50 to 100 km/hr, interaction effect of at least 3 pulses should be considered, and

iii) For a velocity greater than 100km/hr, effect of at least 5 pulses is necessary in summation.

19. Ratios of maximum and minimum value of contact force to the total weight of the vehicle have been plotted against length of pulse for three velocities in Figs.4.26 and 4.27. Length of pulse varies from 1.5m to 7.5m.

Corresponding variation for h is 5mm to 25mm. These ratios have been plotted to determine distribution functions of F_{max} and F_{min} when probability density function of l is uniform. It is seen that the increase velocity increases peak value of max. contact force. While reverse effect is seen in the case of min contact force.

20. Distribution functions of $F_{max}/(M+m)g$ and $F_{min}/(M+m)g$ have been plotted in figs. 4.28 and 4.29, for a velocity of 60km/hr. Range of $F_{max}/(M+m)g$ is 1.27 to 1.78 and $F_{min}/(M+m)g$ is 0.42 to 0.95. It is seen that the probability of getting $F_{max}/(M+m)$ greater than 0.50 is 0.80 while probability of getting $F_{min}/(M+m)g$ greater than 0.9 is 0.05 only.

21. Fatigue calculations: Fatigue failure is the result of cumulative damage that arises when response of structure to external excitation fluctuates. As an application of the above analysis to the fatigue failure of the joint, let us consider one km. length of the track, the number of joints or load repetitions will be

$$= 1000/13 \approx 77$$

Since joint spacing is 13 meter

Let us assume the design dynamic load factor (DLF) as = 1.77 for each type of unevenness.

(DLF) can be expressed in terms of error function.

For continuous unevenness case, the mean value of contact force is the weight of the vehicle. F= (M+m) g. Also r.m.s. value of DLF at a velocity of 60 km/hr = F = 0.1375 (M+m) g.

There fore -

P(DLF 1.77) = Probability of DLF or F_{max} / (M+m) g less than or equal to 1.77.

$$= 0.5 + \text{erf} \ \frac{1.77 - (\ _{F}/(M+m) \ g)}{(\ _{F}/(M+m) \ g \ )}$$

$$= 0.5 + \text{erf} \ \frac{1.77 - 1}{0.1375}$$

$$= 1.00$$

hence probability of DLF 1.77 (probability of failure) is zero.

(iii) <u>Total Unevenness</u> :

Superimposing the two types of unevenness, the probability of design load factor greater than 1.77

= probability of DLF 1.77 for descrete unevenness + probability of DLF 1.77 for continuous unevenness.

= 0.96 + 0.0

= 0.96

Hence probability of failure for design DLF = 1.77 is 0.96.

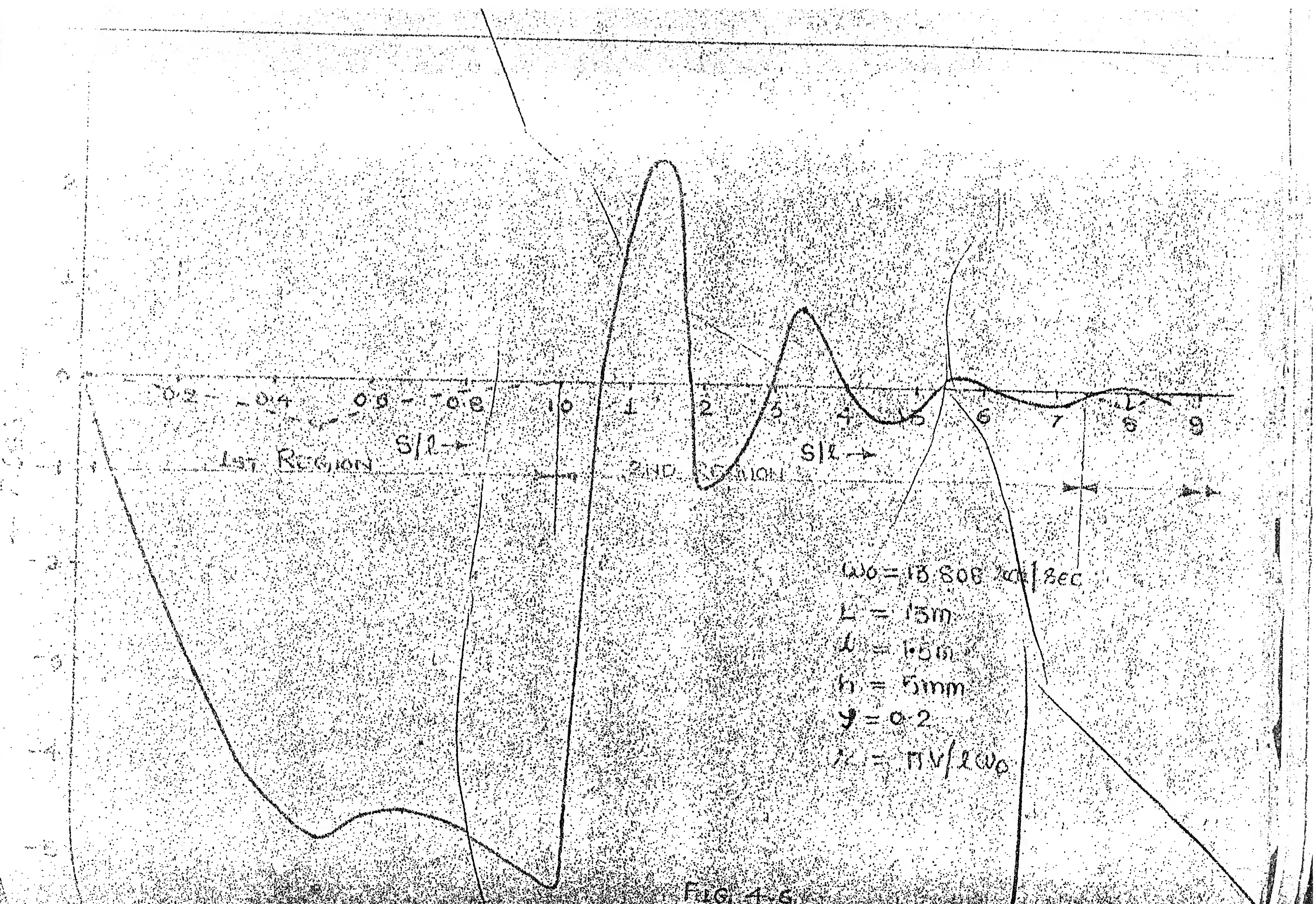

1st Region S/l →
2nd Region S/l →
ω0 = 13.808 rad/sec
L = 15m
l = 1.5m
h = 5mm
γ = 0.2
λ = πV/2ω0

FIG. 4.6

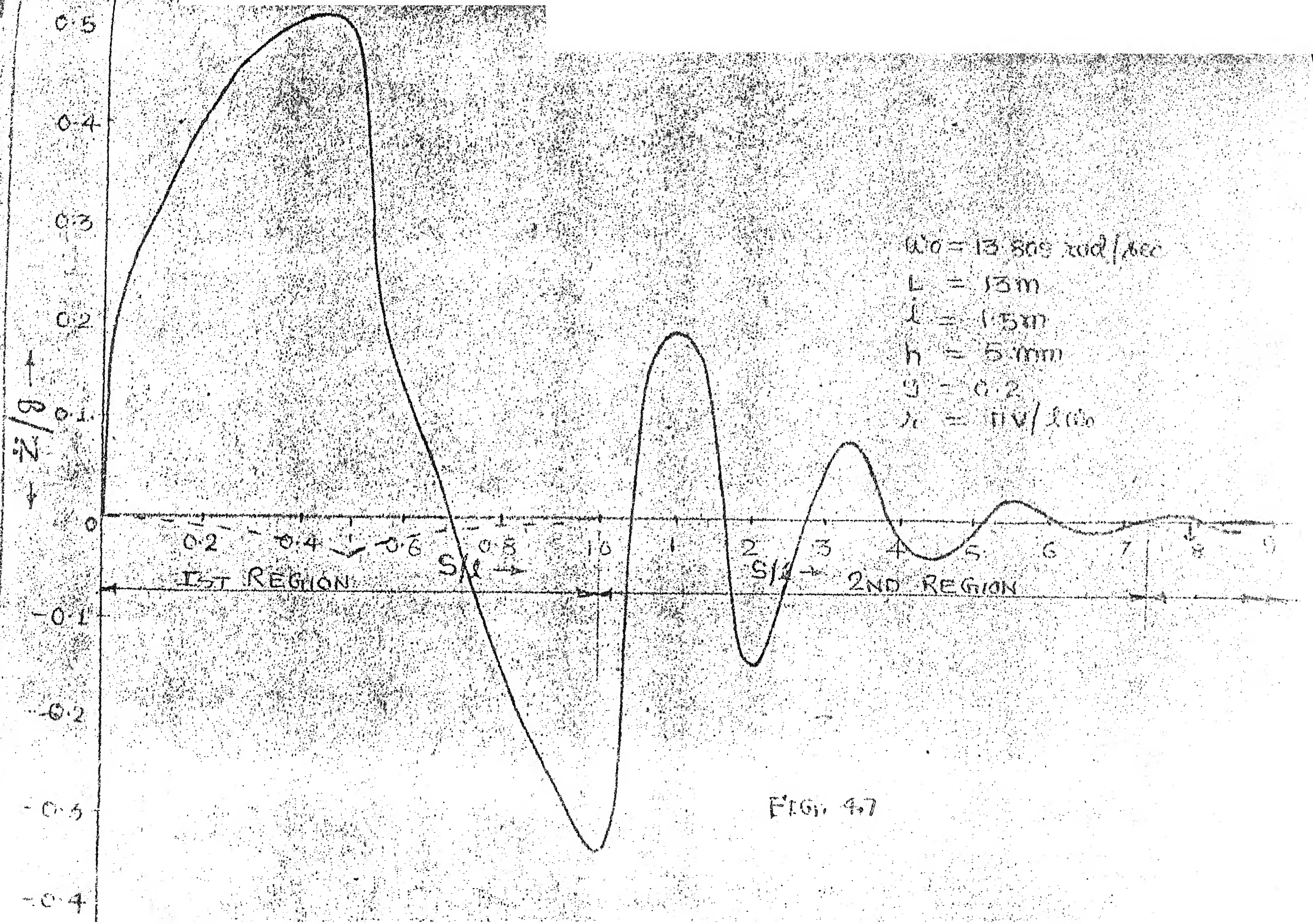
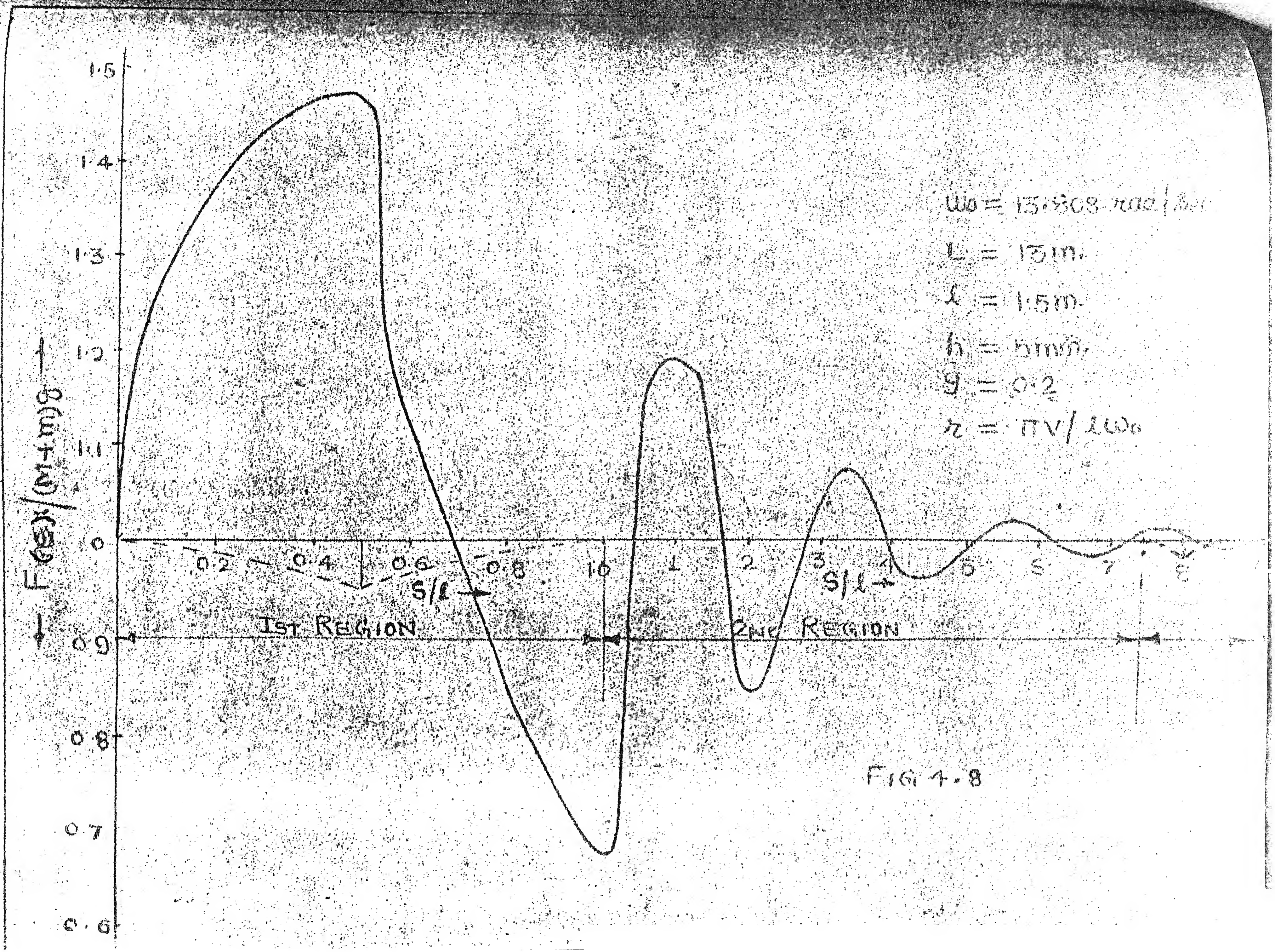
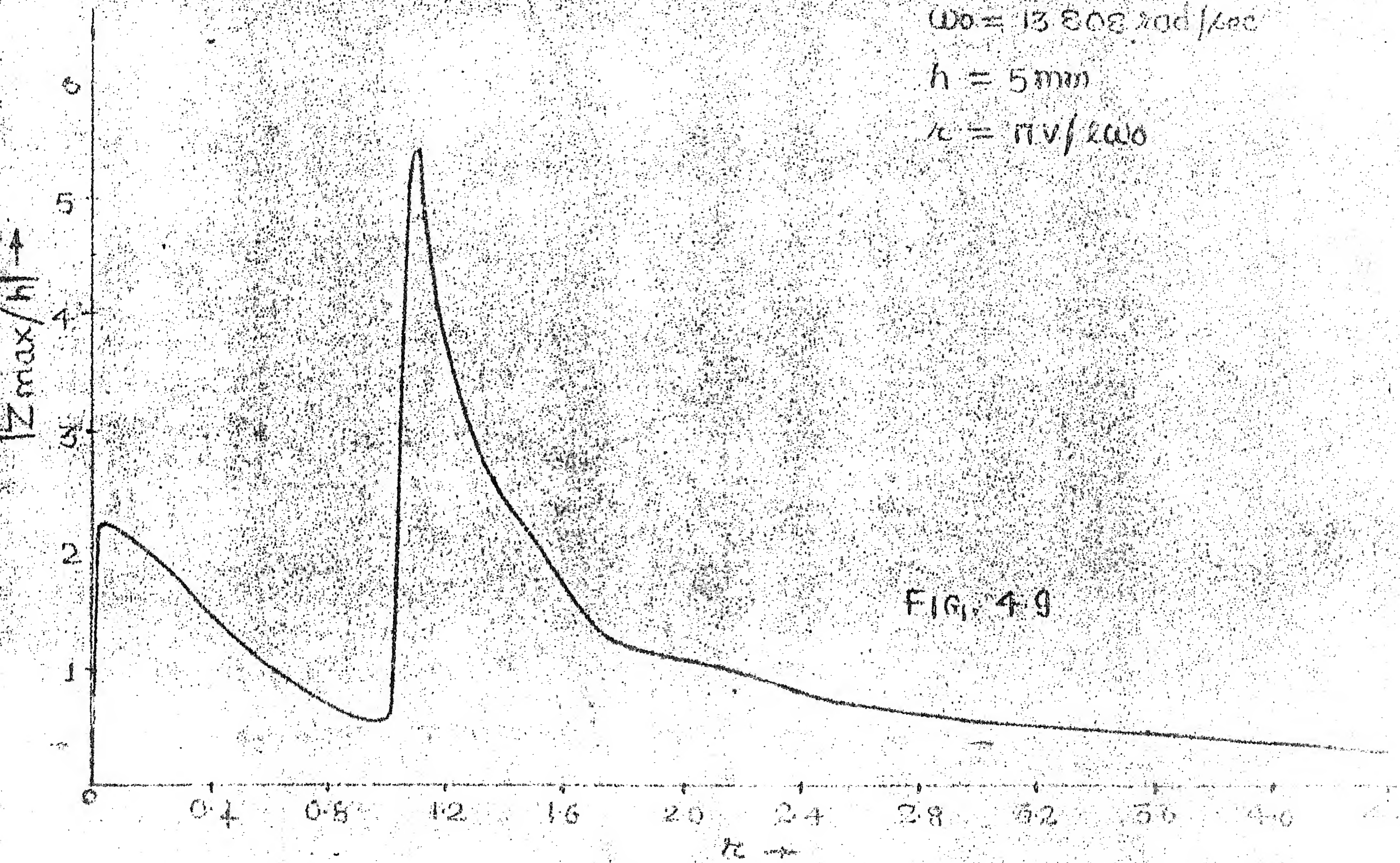

0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
$\ddot{Z}/g$
0.2
0.4
0.6
0.8
1.0
1
2
3
4
5
6
7
8
9
S/ℓ
S/ℓ
1st REGION
2ND REGION
$\omega_0 = 13.809$ rad/sec
$L = 13$ m
$\ell = 1.5$ m
$h = 5$ mm
$\zeta = 0.2$
$\lambda = \pi V/\ell\omega_0$

FIG. 4.7



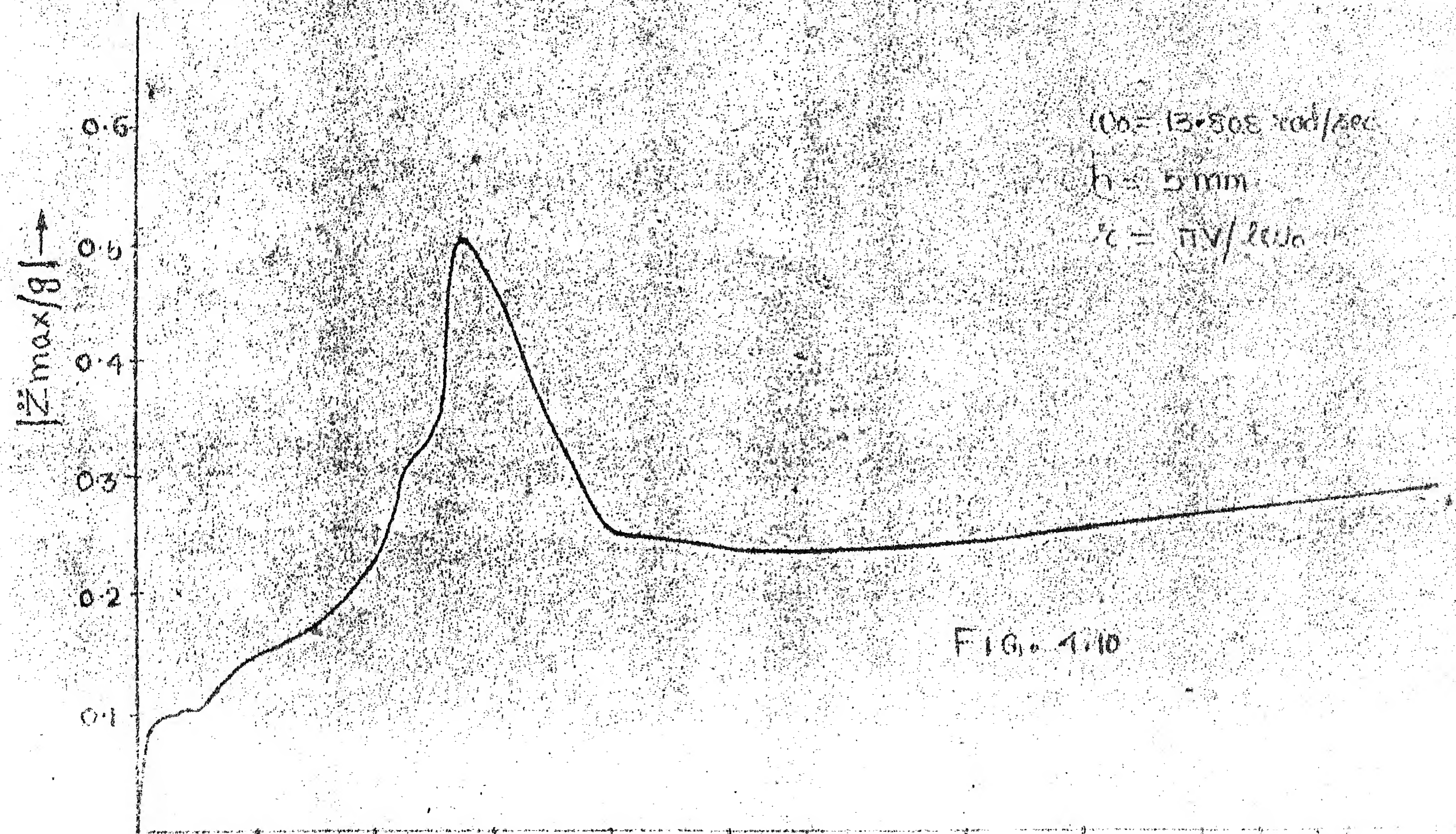
$\omega_0 = 13.808$ rad/sec
$L = 13$ m.
$l = 1.5$ m.
$h = 5$ mm.
$g = 0.2$
$r = \pi V / l\omega_0$
$F(s)/(M+m)g$
S/l
1st Region
2nd Region

FIG 4.8



ωo = 13 808 rad/sec
h = 5 mm
r = πv/lωo
|Zmax/h| →
r →
0
0·4
0·8
1·2
1·6
2·0
2·4
2·8
3·2
3·6
4·0
1
2
3
4
5
6

FIG. 4·9



$\omega_0 = 13.508$ rad/sec
$h = 5$ mm
$\lambda = \pi V / 2\omega_0$
$|\ddot{Z}_{max}/g|$
0.6
0.5
0.4
0.3
0.2
0.1
0
0.4
0.8
1.2
1.6
2.0
2.4
2.8
3.2
3.6
4.0
λ

FIG. 4.10

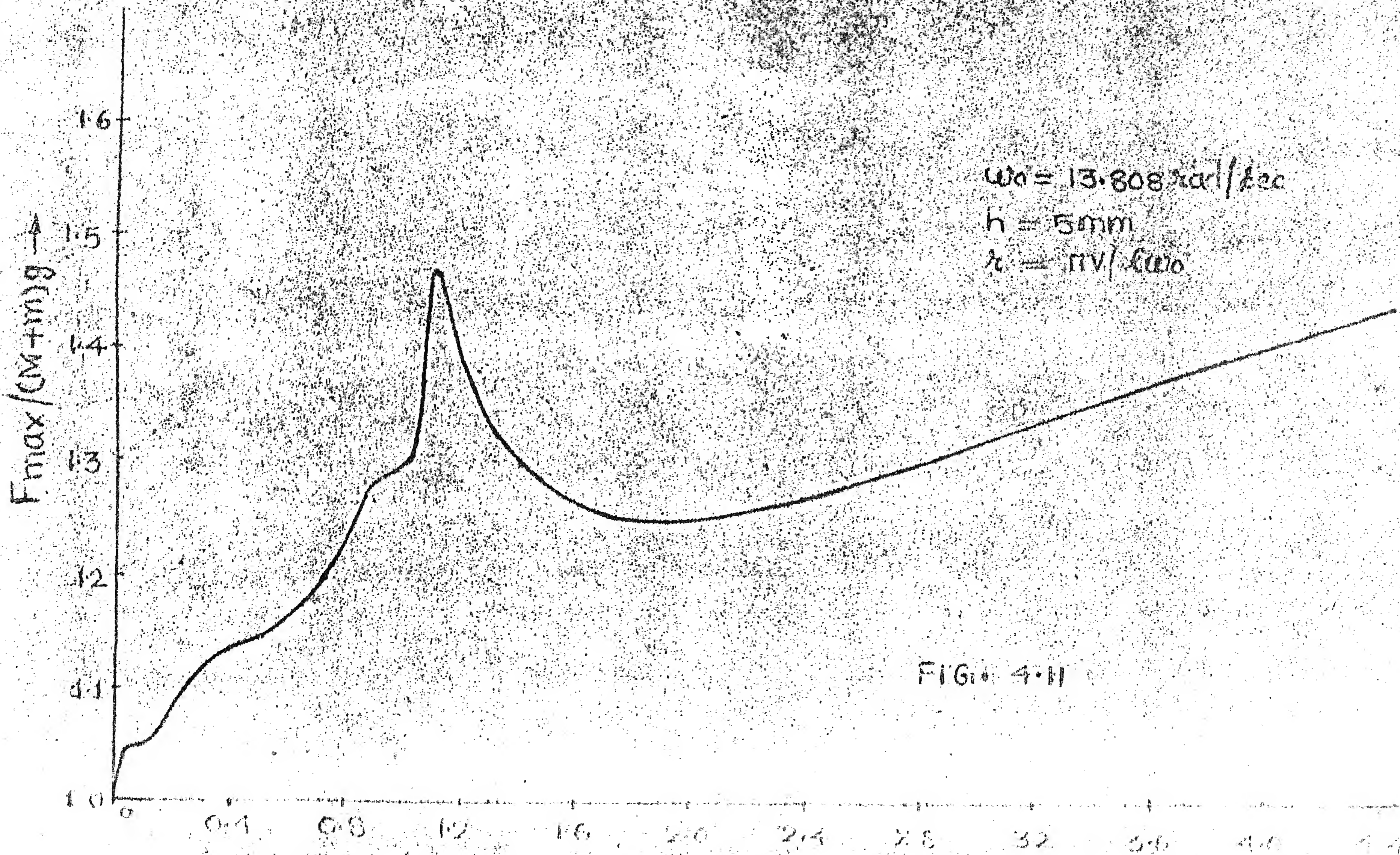
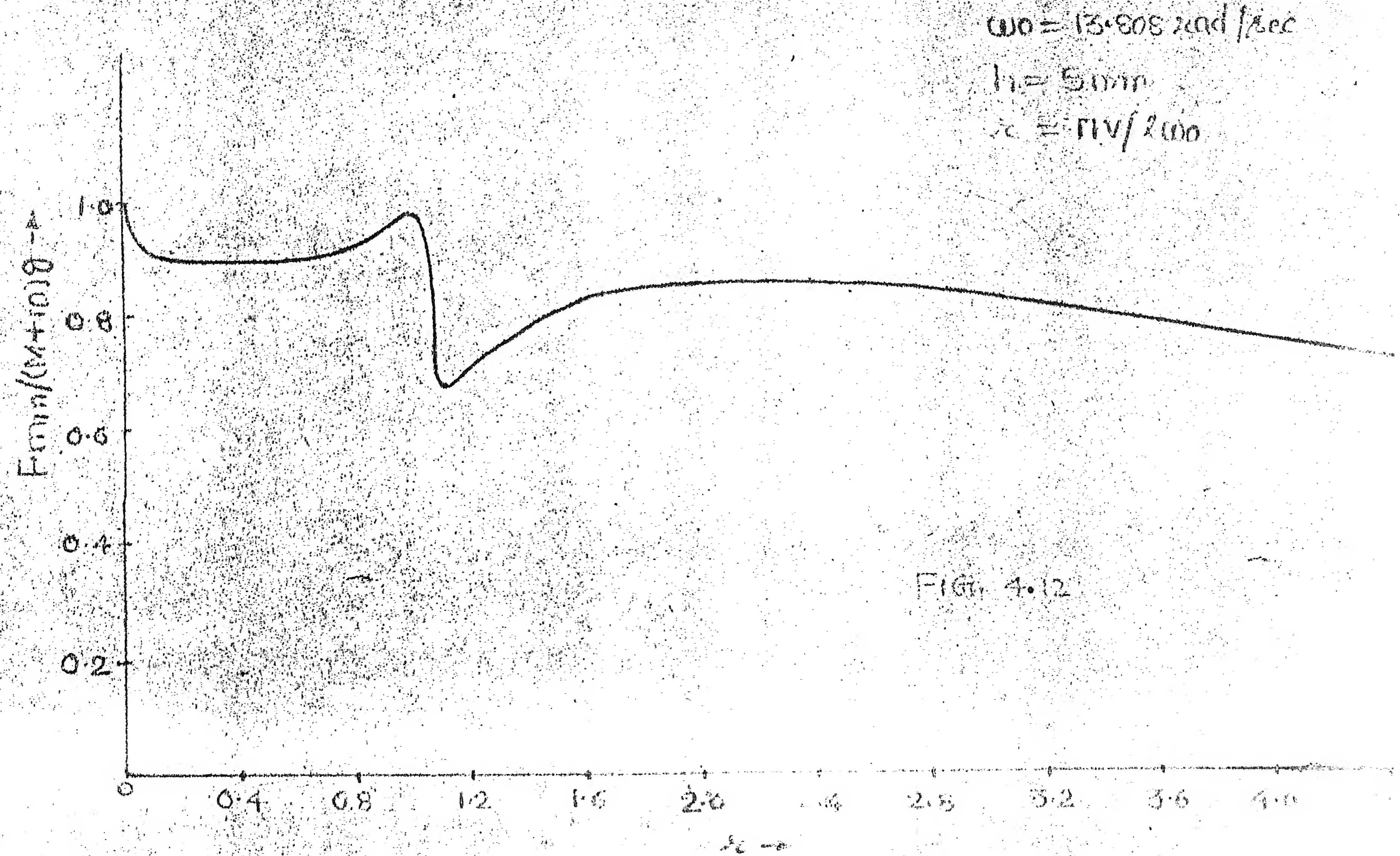
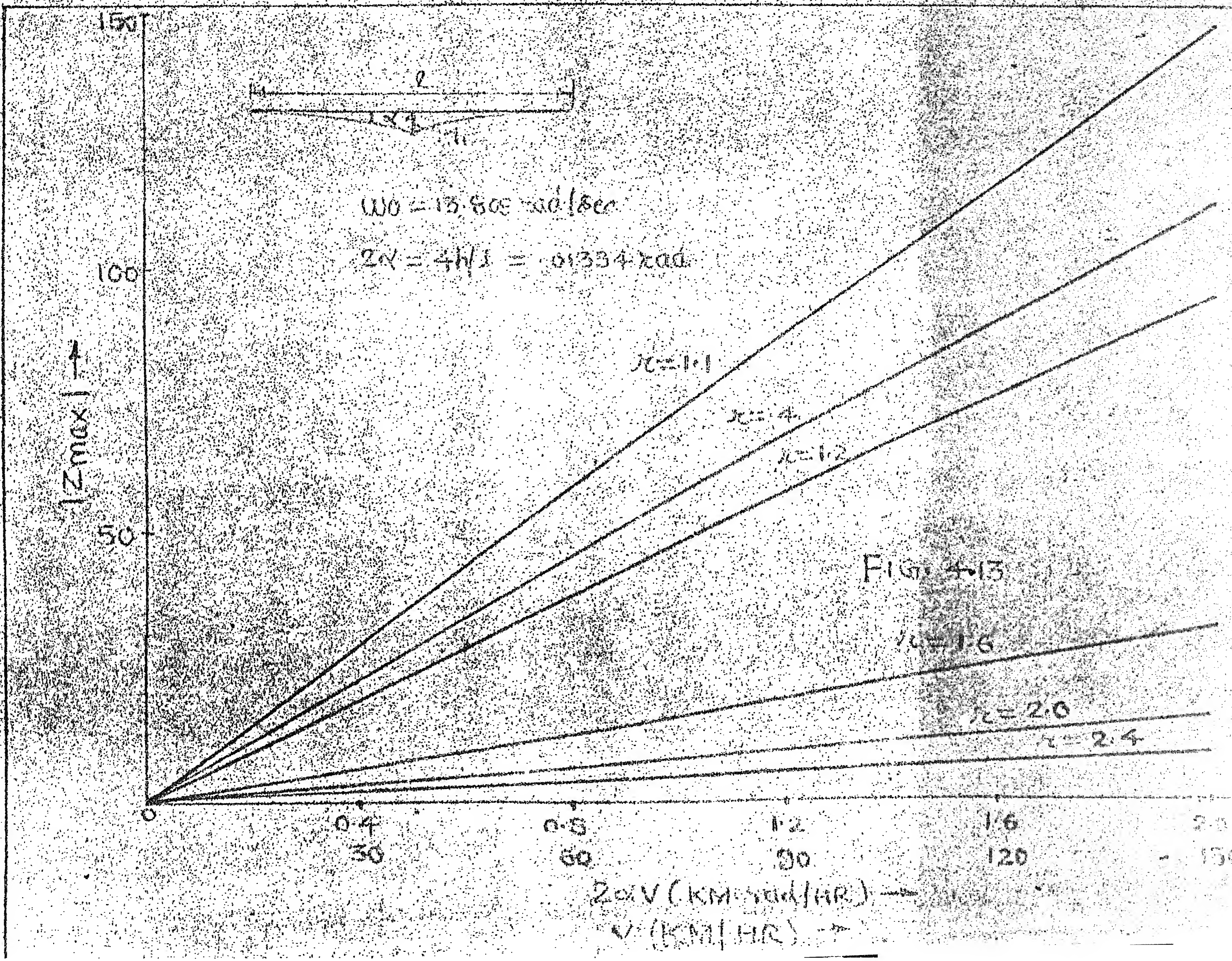

$\omega_0 = 13.808$ rad/sec
h = 5mm
$\lambda = \pi v / l\omega_0$
Fmax/(M+m)g →
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0
0.4
0.8
1.2
1.6
2.0
2.4
2.8
3.2
3.6
4.0
4.4

FIG. 4·11



ωo = 13·808 rad/sec
h = 5 mm
x = nv/2ωo
Fmn/(M+m)g →
1·0
0·8
0·6
0·4
0·2
0
0·4
0·8
1·2
1·6
2·0
2·8
3·2
3·6
x →

FIG. 4·12



l
h
ω0 = 13.809 rad/sec
2α = 4h/l = .01334 rad
150
100
50
0
|Zmax| →
λ=1.1
λ=.4
λ=1.2
λ=1.6
λ=2.0
λ=2.4
0.4
0.8
1.2
1.6
2.0
30
60
90
120
150
2αV (KM rad/HR) →
V (KM/HR) →

FIG. 4.13

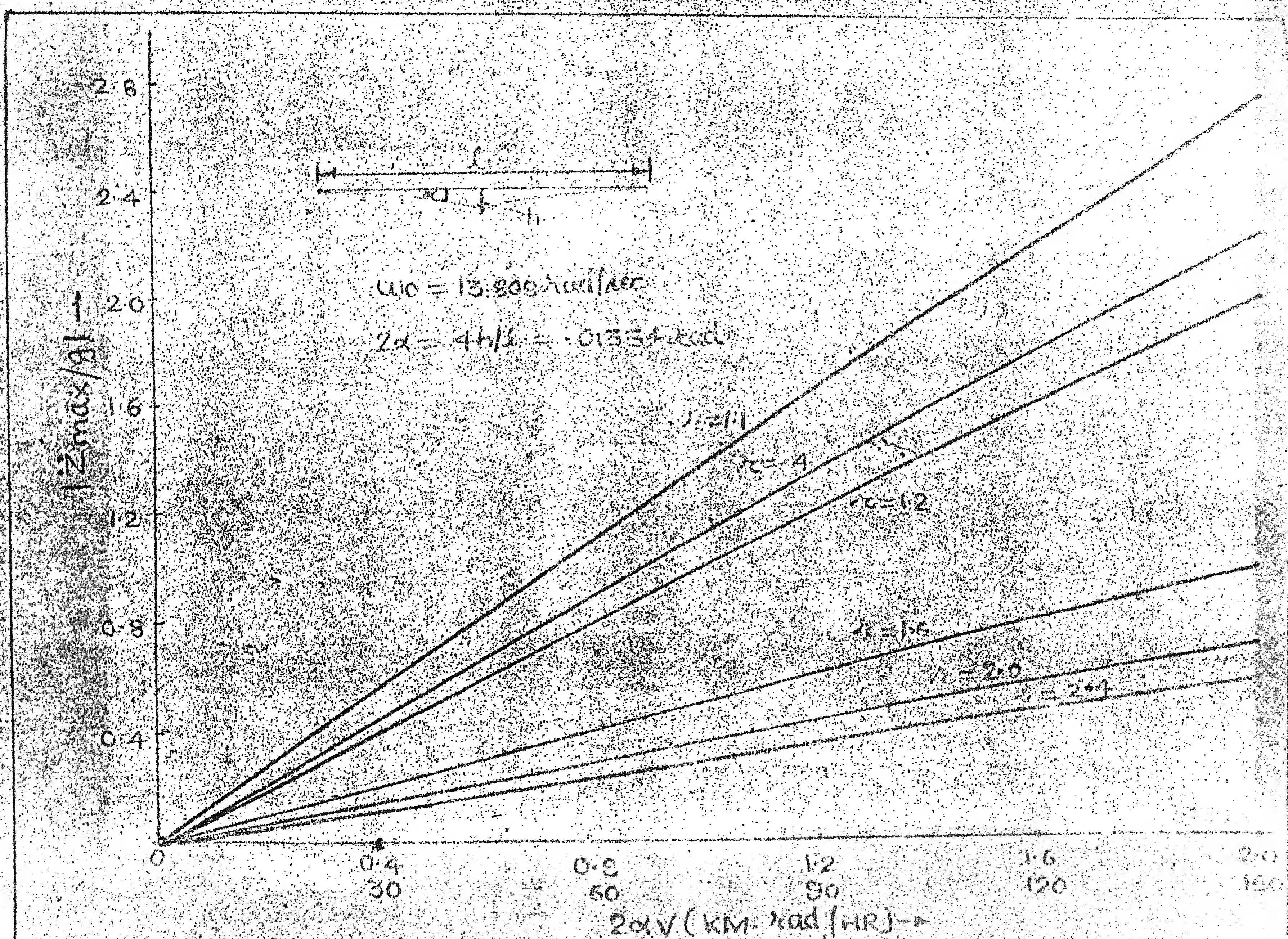
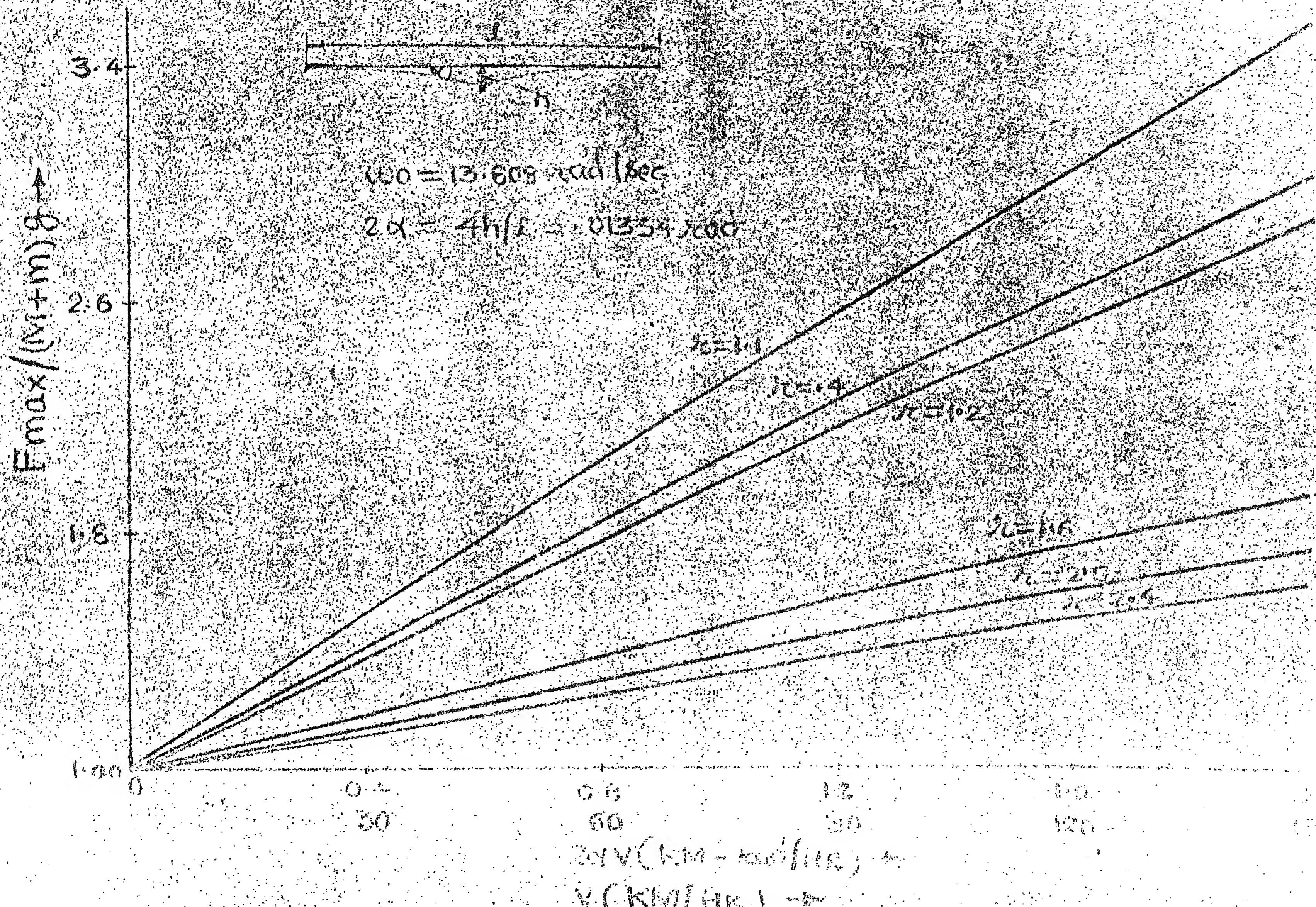
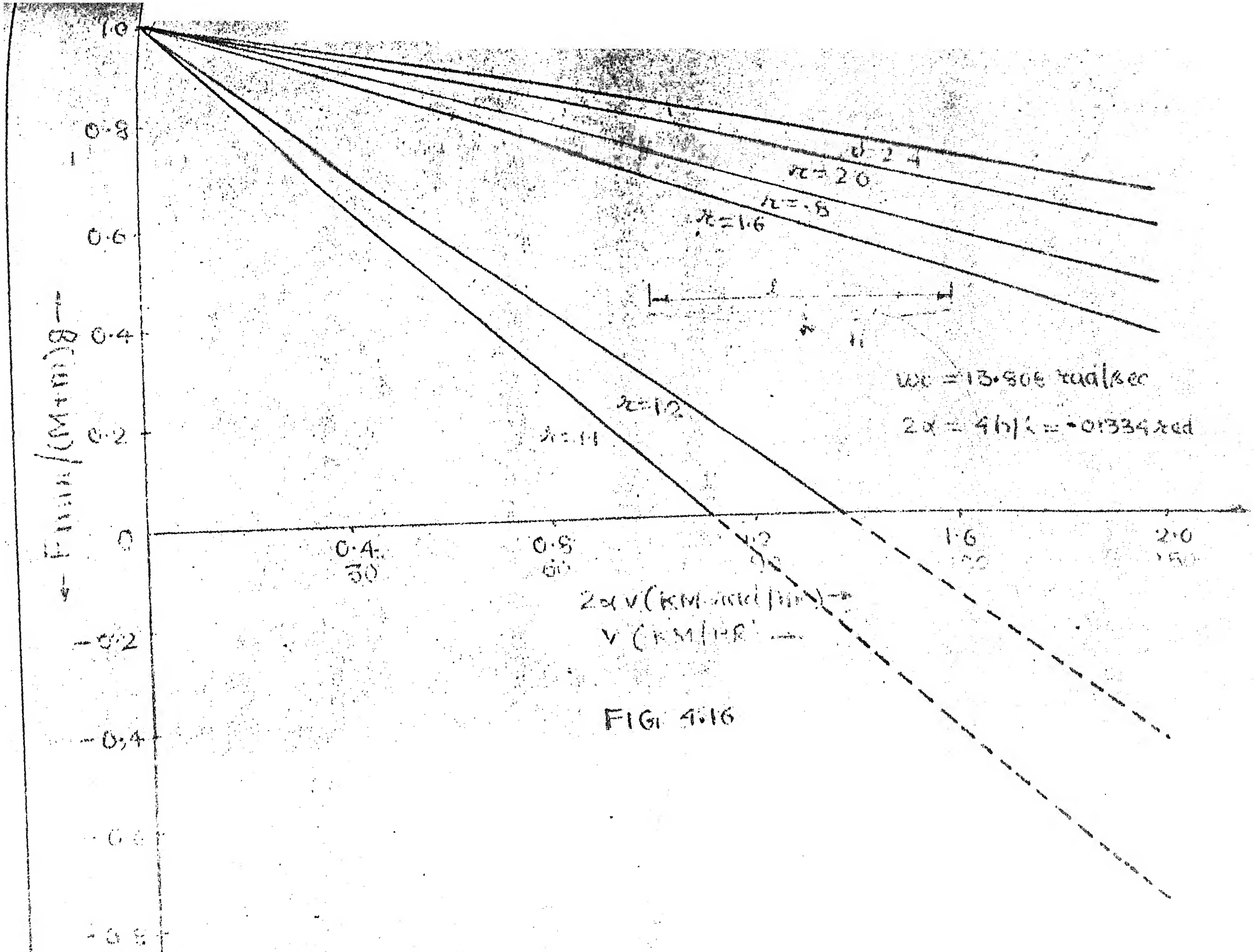

2.8
2.4
2.0
1.6
1.2
0.8
0.4
0
0.4
30
0.8
60
1.2
90
1.6
120
2.0
150
$|\ddot{Z}max/g|$
$\omega_{10} = 13.809$ rad/sec
$2\alpha = 4h/\ell = .0134$ rad
2αV (KM rad/HR)
V (KM/HR)
τ = 4
τ = 1.2
τ = 1.6
τ = 2.0
τ = 2.4

FIG. 4.14



ℓ
α
h
$\omega_0 = 13.608$ rad/sec
$2\alpha = 4h/\ell = .01354$ rad
$F_{max}/(M+m)g \rightarrow$
3.4
2.6
1.8
1.00
0
0.4
0.8
1.2
1.6
30
60
90
120
$2\alpha V$ (KM - rad/HR) →
V (KM/HR) →

FIG. 4.14



1.0
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
0.4
30
0.8
60
1.2
1.6
2.0
$\leftarrow F_{max}/(M+m)g \rightarrow$
$2\alpha v$ (KM-rad/hr) →
v (KM/HR) →
$\omega c = 13.906$ rad/sec
$2\alpha = 4h/l = .01334$ rad
$x = 2.0$
$x = 1.8$
$x = 1.6$
$x = 1.2$
$x = 1.1$

FIG. 4.16

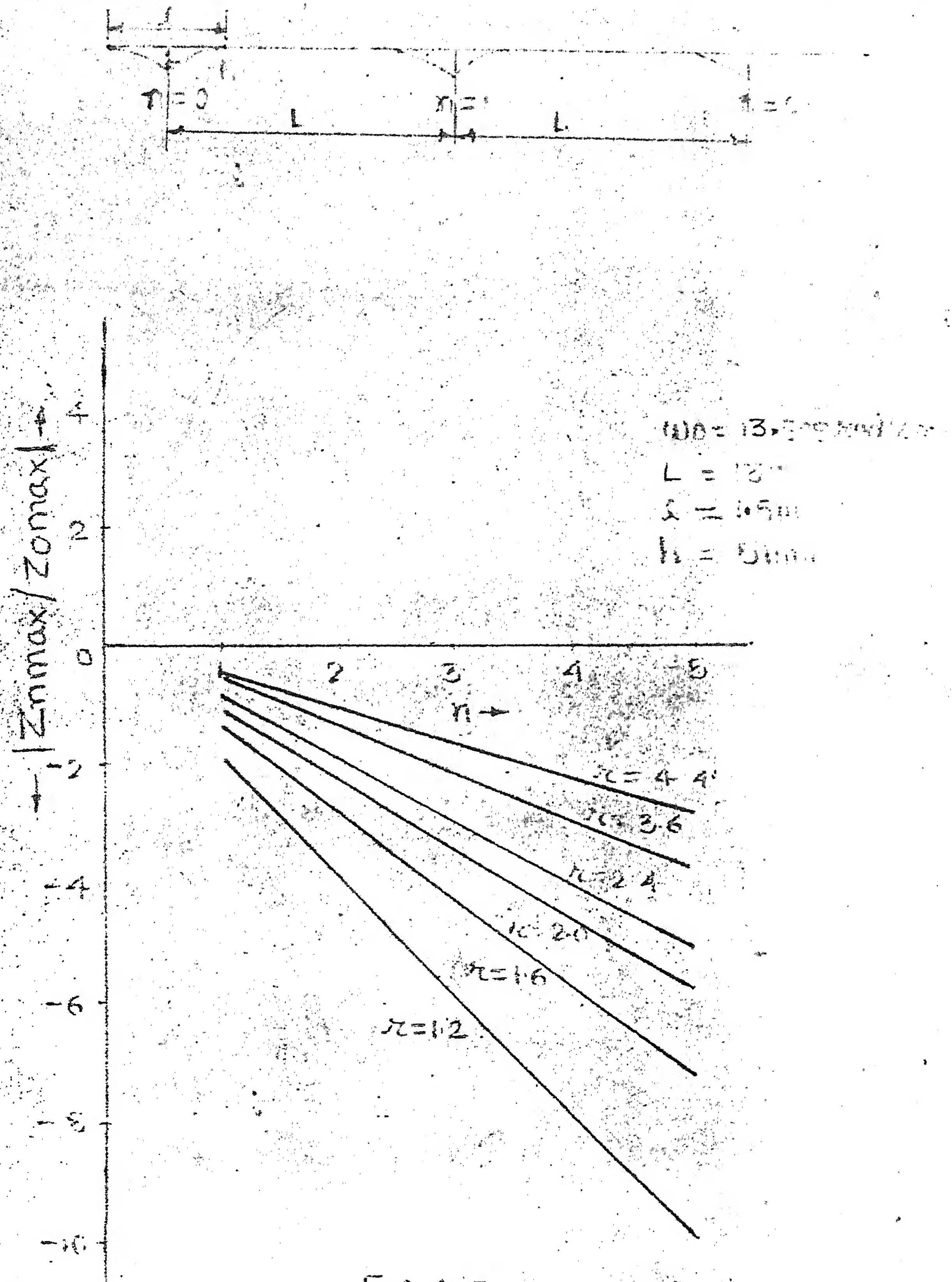


FIG 4.17

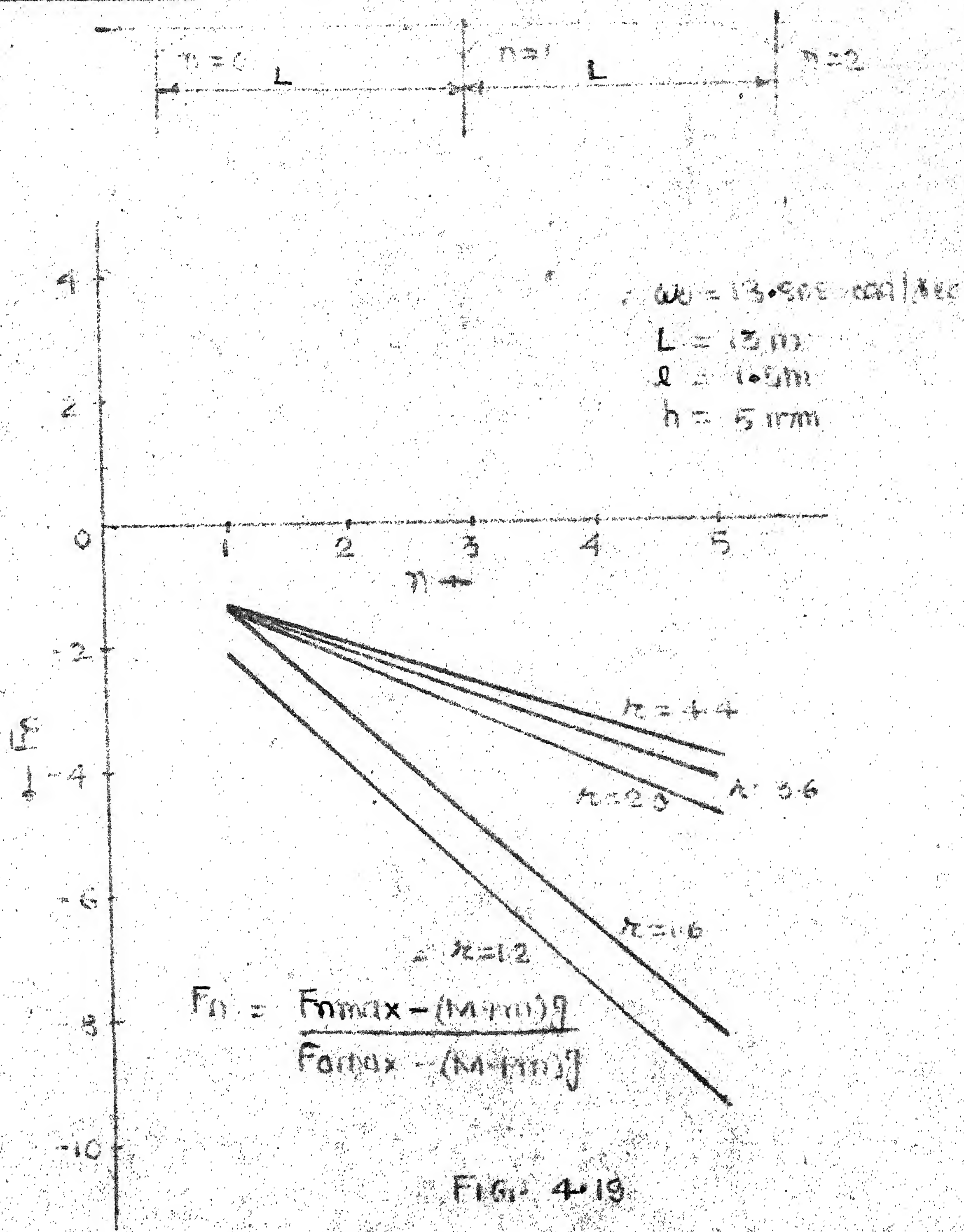
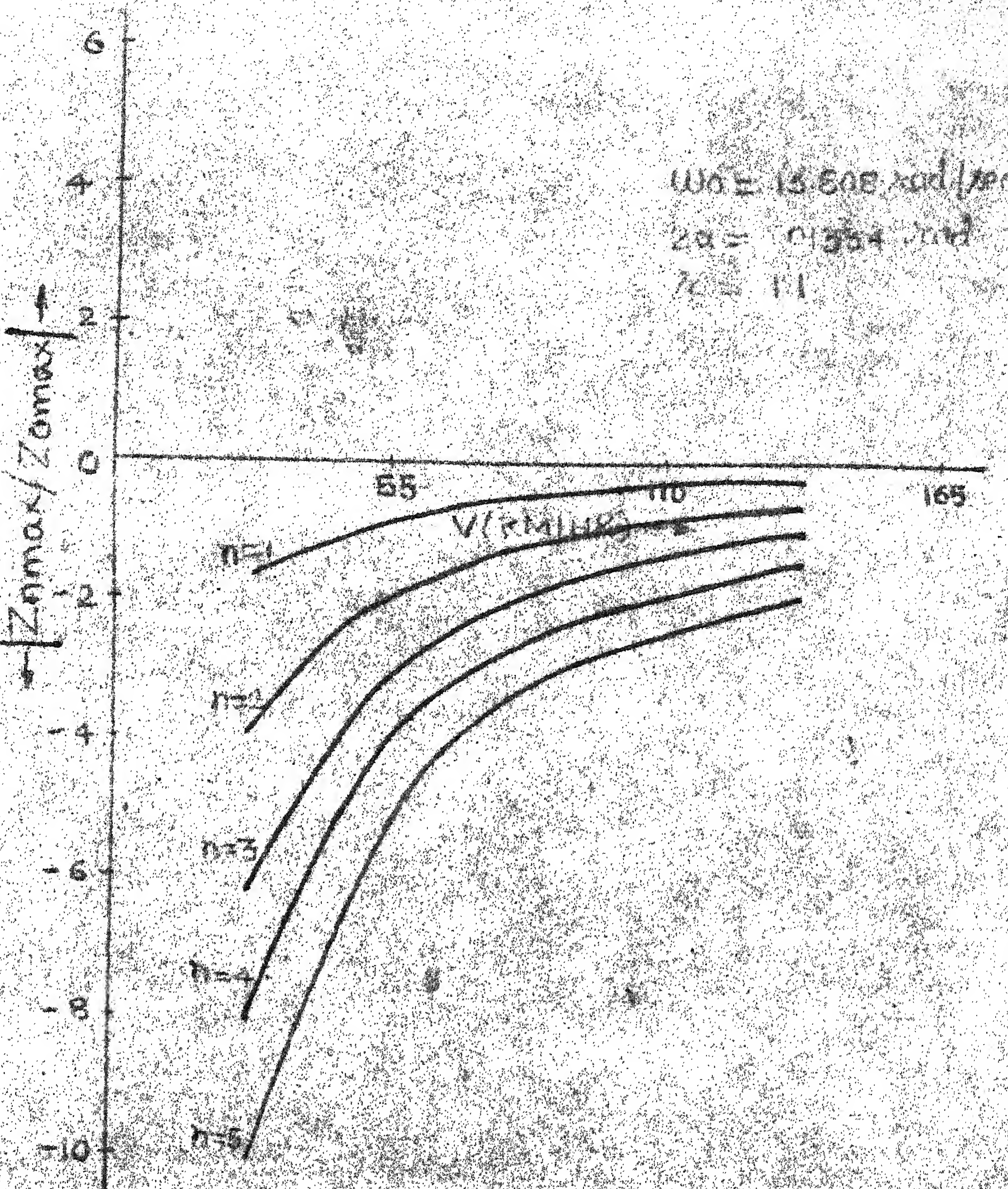

n=0
L
n=1
L
n=2
ω = 13.968 rad/sec
L = 13 m
l = 1.5 m
h = 5 mm
0
1
2
3
4
5
n →
-2
-4
-6
-8
-10
r=4.4
r=3.6
r=2.8
r=1.6
r=1.2
Fn = Fnmax − (M+m)g / Fomax − (M+m)g

FIG. 4.19



6
4
2
0
-2
-4
-6
-8
-10
55
110
165
$-|Z_{n max}/Z_{o max}|$
V(KM/HR)
n=1
n=2
n=3
n=4
n=5
ω_n = 15.808 rad/sec

FIG. 4.20

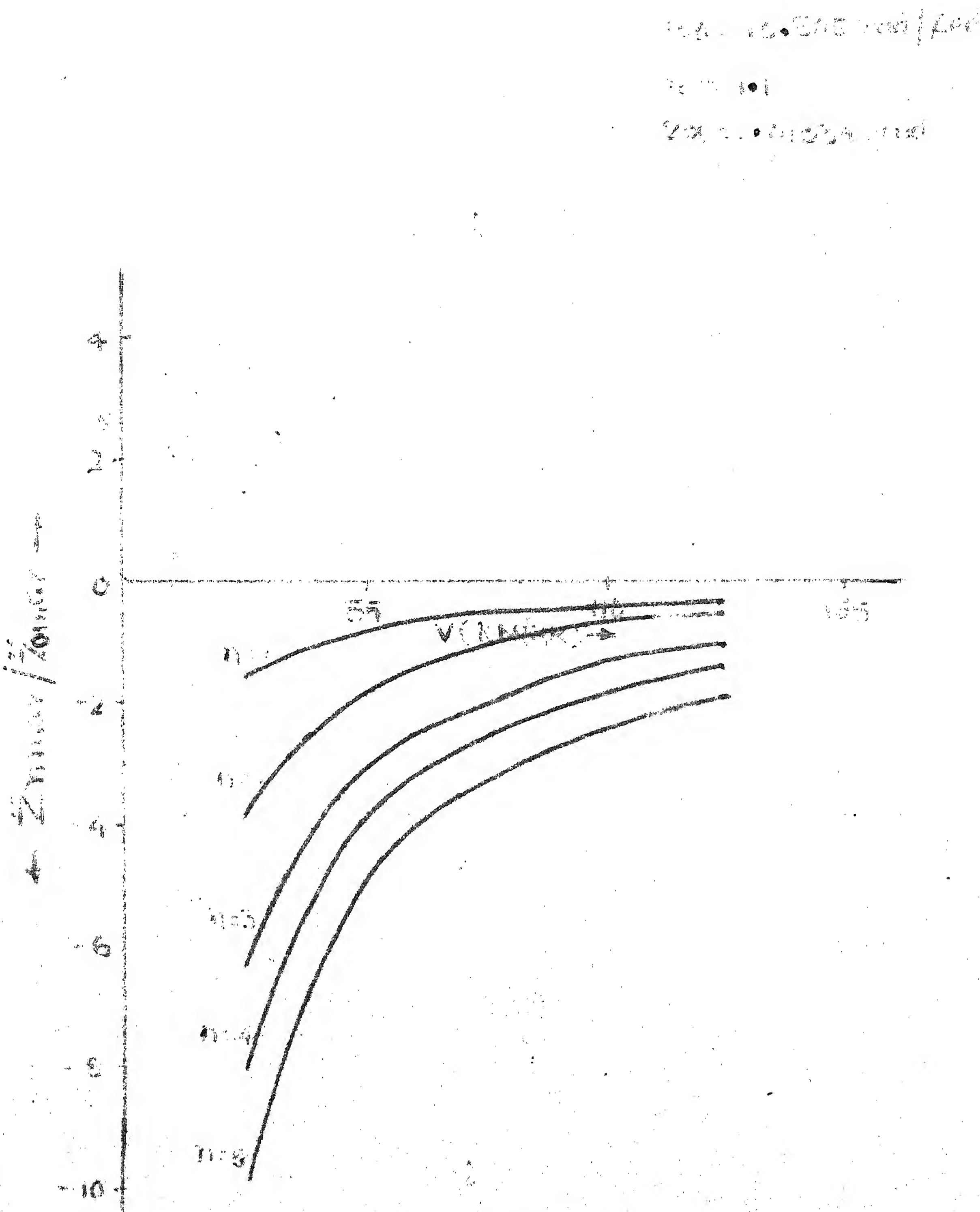
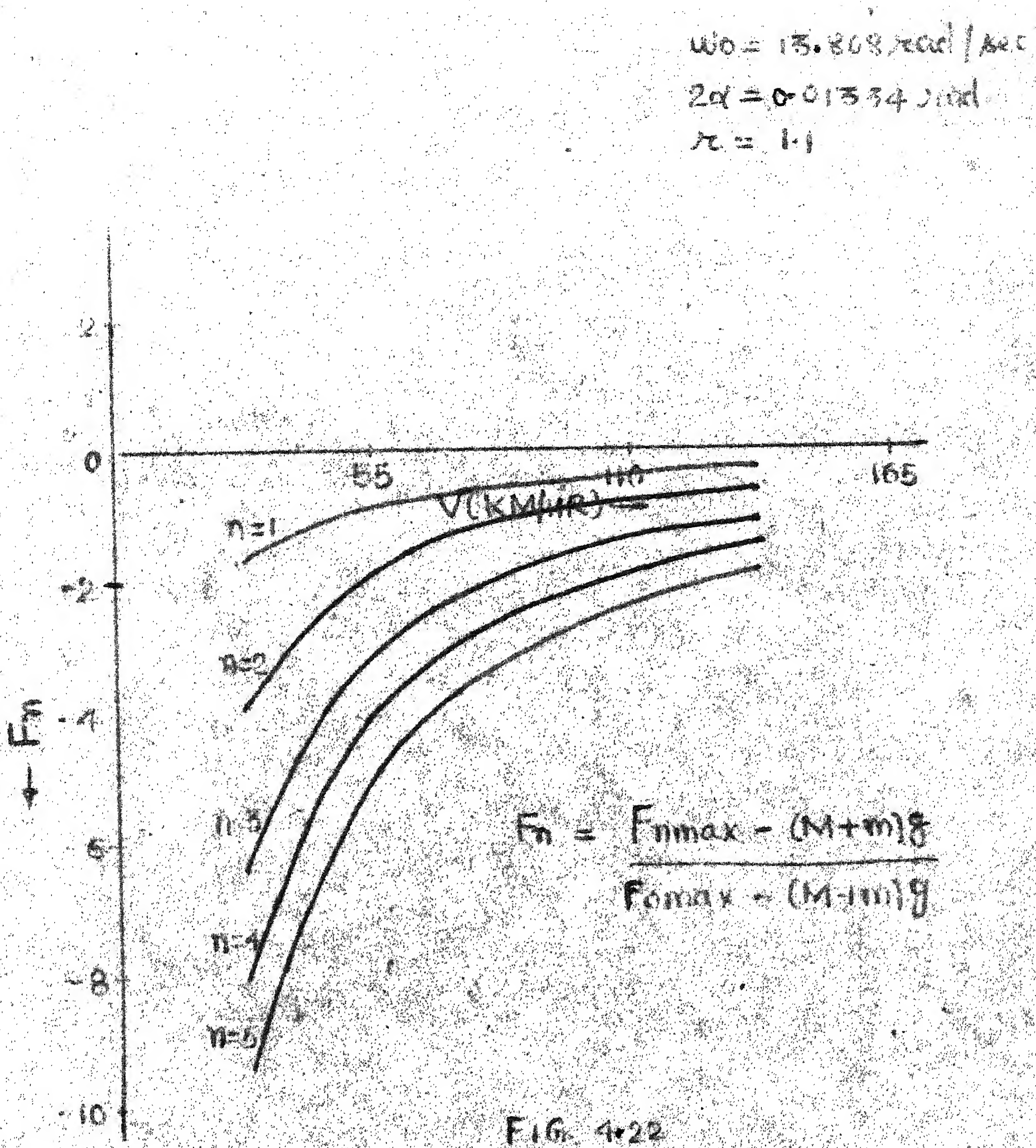


FIG. 4.21



wo = 13.808 rad/sec
2α = 0.01334 rad
r = 1.1
2
0
-2
-4
-6
-8
-10
55
110
165
V(KM/HR) →
n=1
n=2
n=3
n=4
n=5
← Fn
Fn = Fnmax - (M+m)g / Fomax - (M+m)g

FIG. 4.22

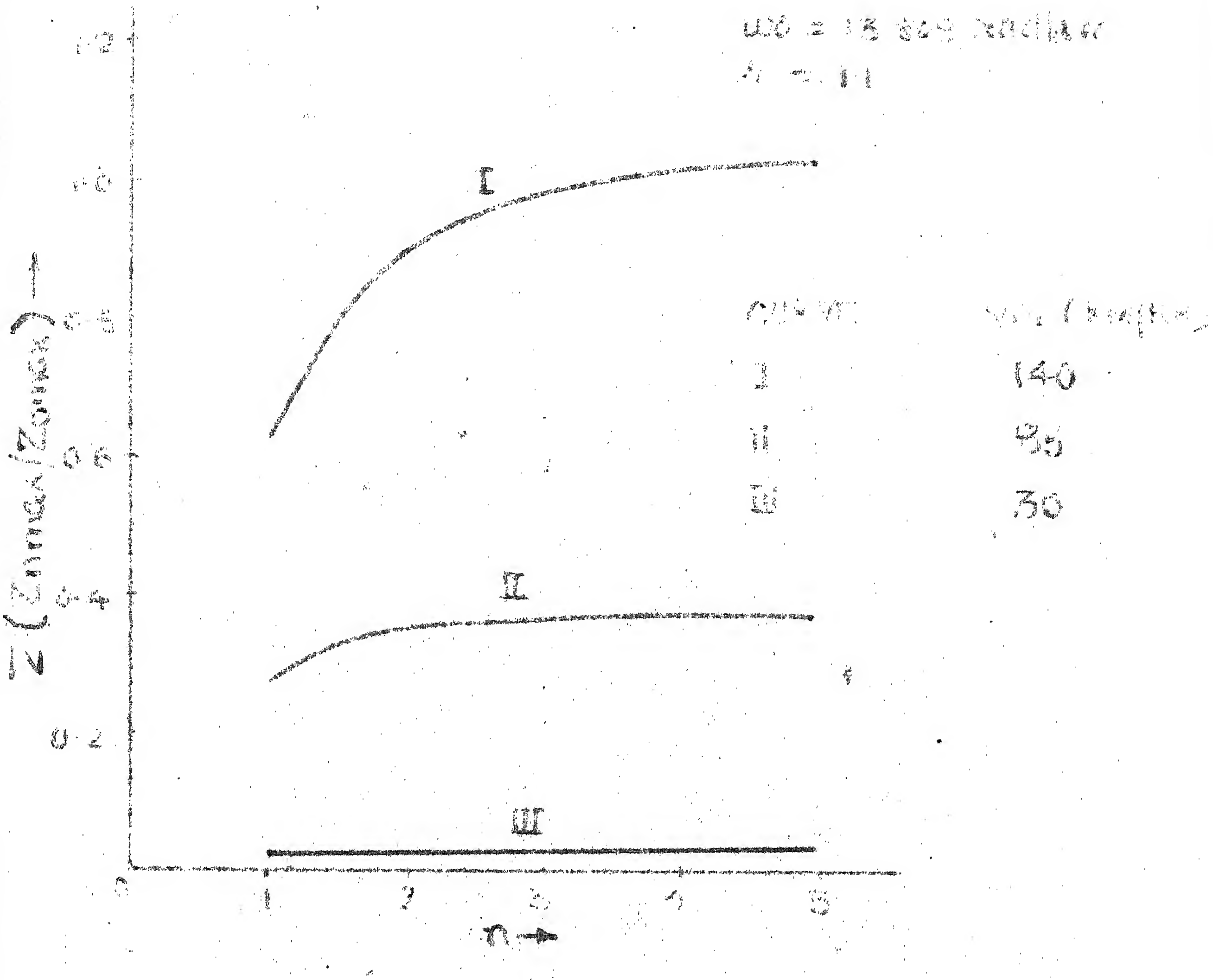


FIG. 4.23

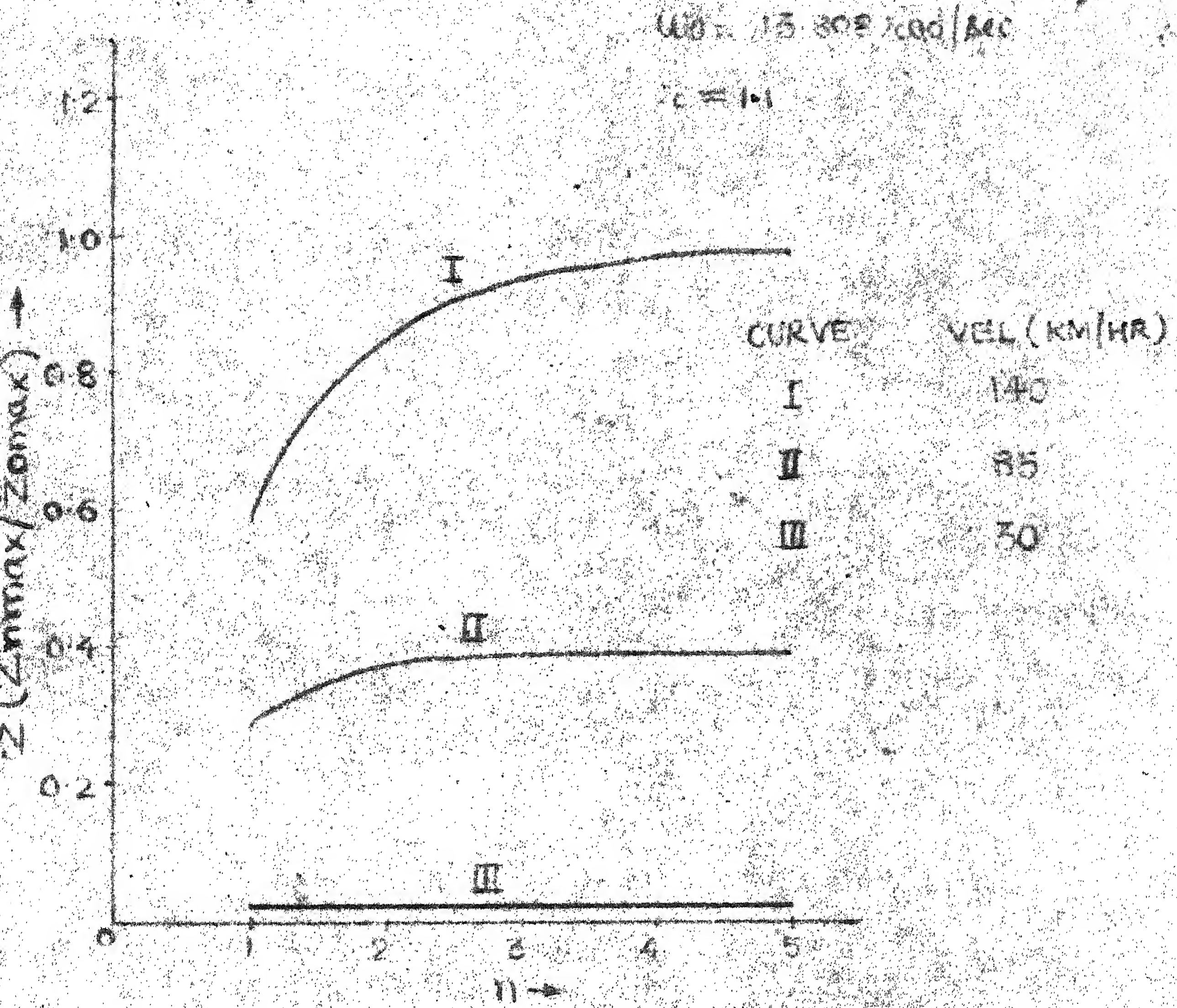
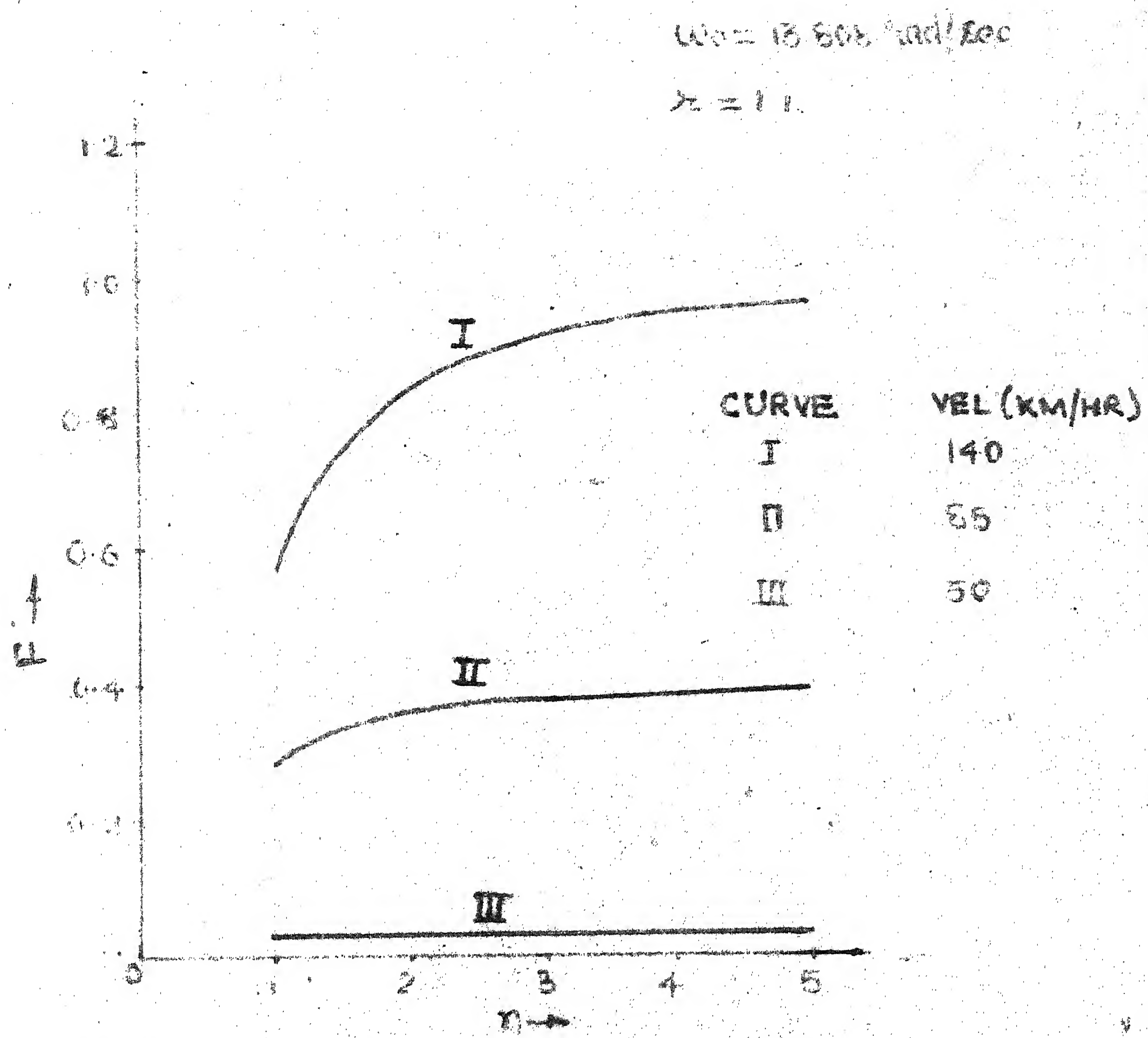

ω0 = 13.308 rad/sec
c = 1.1
1.2
1.0
0.8
0.6
0.4
0.2
0
1
2
3
4
5
n →
Σ(Z̈nmax/Z̈omax) →
I
II
III
CURVE
VEL (KM/HR)
I
140
II
85
III
30

FIG. 4.24



$$F = \left(\sum \left(\frac{F_m max}{(M+m)g} - 1 \Big/ \frac{F_o max}{(M+m)g} - 1 \right) \right)$$

FIG. 4.25

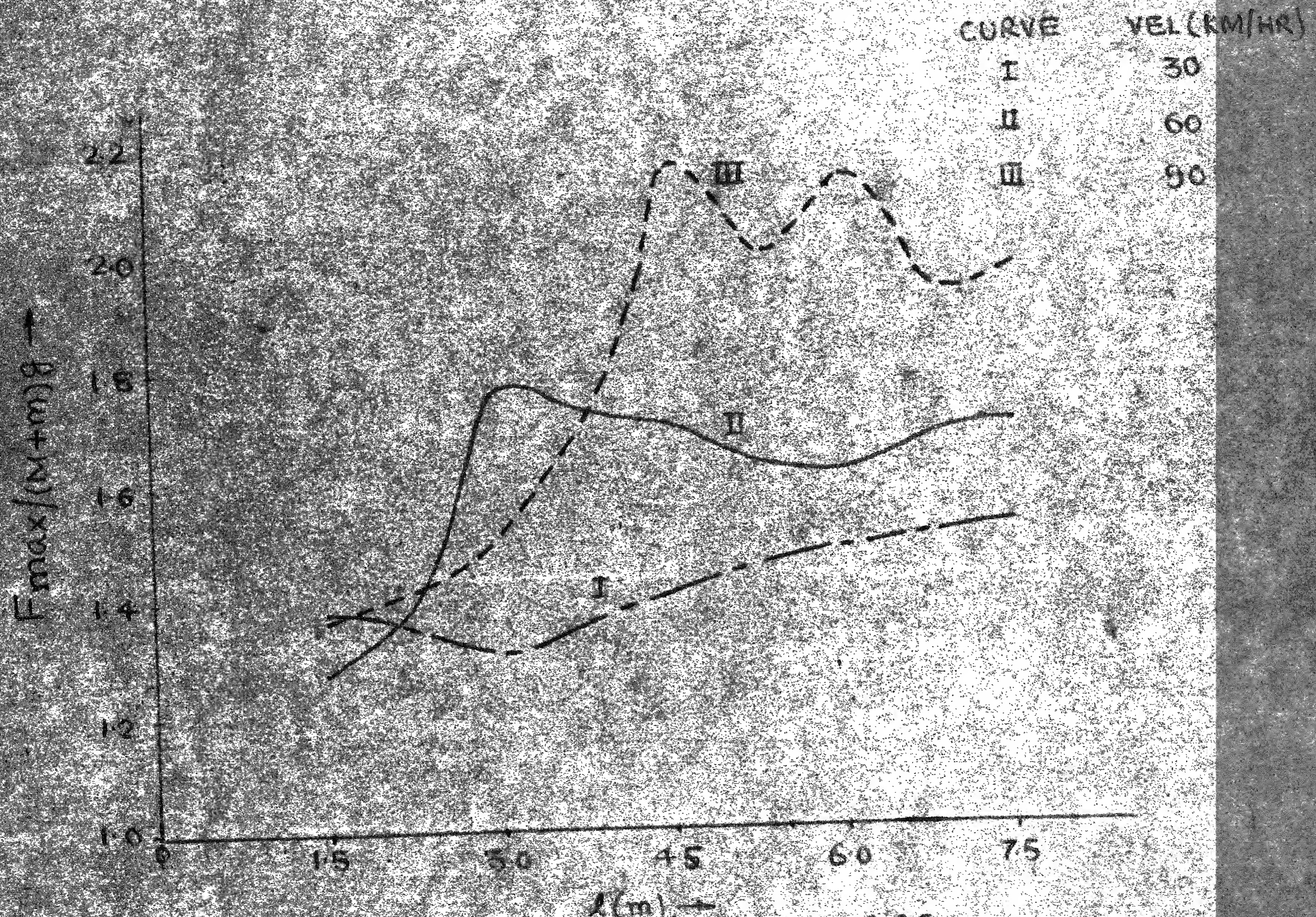

CURVE VEL (KM/HR)
I 30
II 60
III 90
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0
1.5
3.0
4.5
6.0
7.5
$F_{max}/(M+m)g$ →
l(m) →
I
II
III

FIG. 4.26

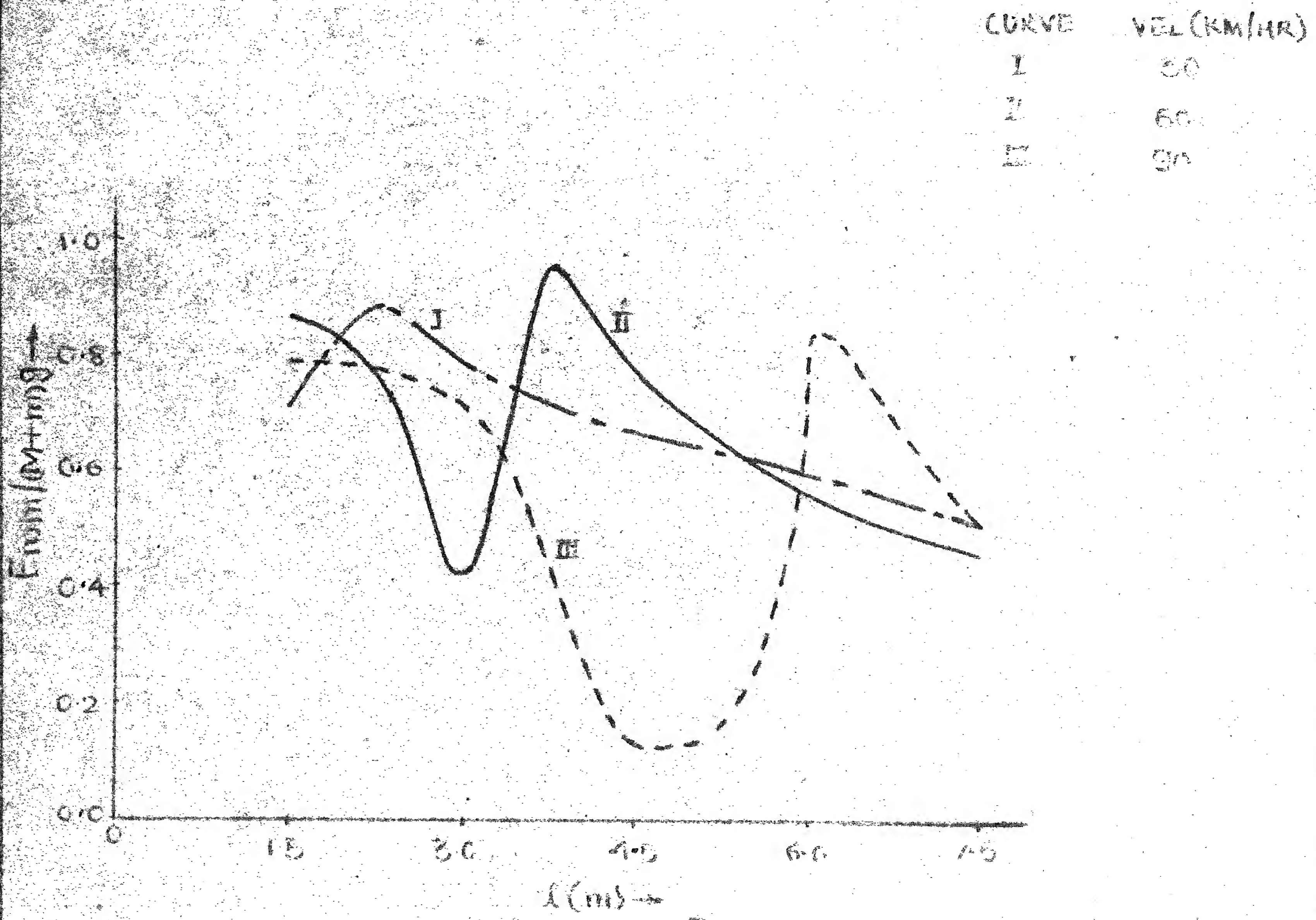

CURVE VEL (KM/HR)
I 30
II 60
III 90
1.0
0.8
0.6
0.4
0.2
0.0
0
1.5
3.0
4.5
6.0
7.5
Fnom/(M+m)g →
ℓ (m) →
I
II
III

FIG 4.17

$P(F_{max}/(M+m)g)$ = Probability Distribution function of $F_{max}/(M+m)g$

VELOCITY = 60 KM/HR

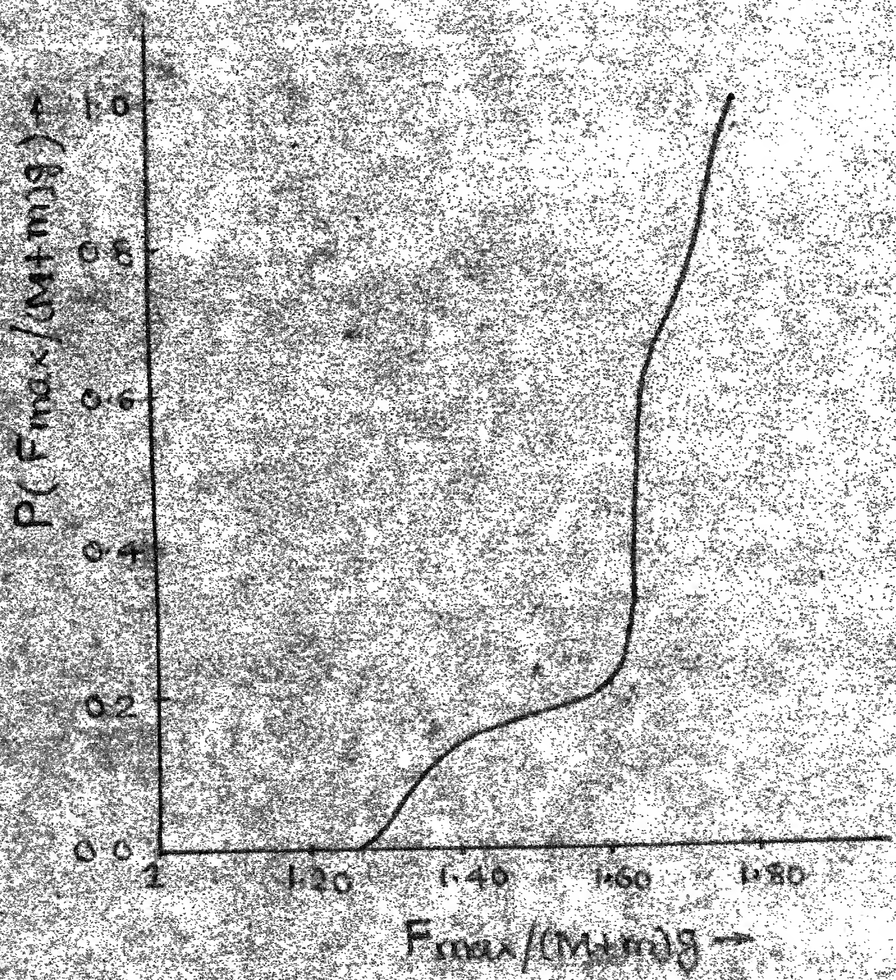
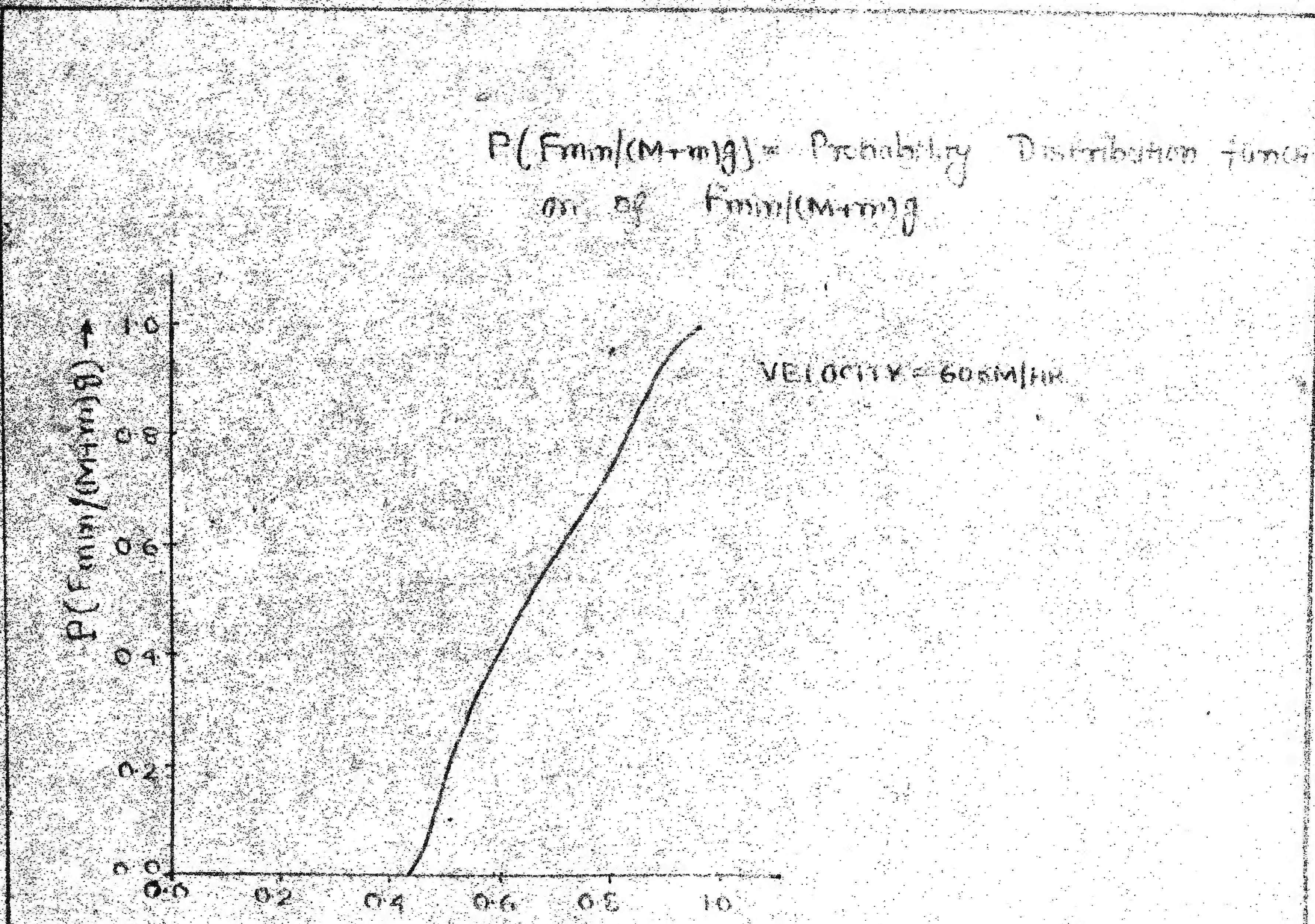


FIG. 4.28



P(Fmin/(M+m)g) = Probability Distribution function of Fmin/(M+m)g
VELOCITY = 60KM/HR
P(Fmin/(M+m)g) →
0.0
0.2
0.4
0.6
0.8
1.0
Fmin/(M+m)g →

FIG. 4.29

TABLE 4.1

1.	Length (L) between two consecutive joints	=	13 meters
2.	Natural frequency (ω_0)	=	13.808 rad/sec.
3.	Length (l) of the joint	=	15 meter
4.	Depth(h) of the joint	=	5 mm
5.	Damping ratio (ζ)	=	0.2
6.	Velocity range	=	0 to 110 km/hr.
7.	Min. value of l	=	1.5meter
8.	Max. value of l	=	7.5 meter
9.	Min. value of h	=	5 mm
10.	Max. value of h	=	25mm

4.21 Fourier Analysis

In the last section, we have calculated response parameters by first considering single pulse and then superimposing response due to each pulse. Since occurrences of these pulses are periodic, response paramters due to series of pulses can be obtained by considering equivalent Fourier series for the excitation process. Fourier constants are calculated and response parameters are obtained by solving convolution integral. Results of Fourier series can be compared with the results obtained in previous section.

Fourier series for excitation Y(S) can be written as (Fig. 4.5).

$$Y(S) = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{2n\pi S}{L} + b_n \sin \frac{2n\pi S}{L} \qquad 4.54$$

where, $$a_o = \frac{2}{L} \int_0^L Y(S)\, dS \qquad 4.55$$

$$a_n = \frac{2}{L} \int_0^L Y(S) \cos \frac{2n\pi S}{L}\, dS \qquad 4.56$$

$$b_n = \frac{2}{L} \int_0^L Y(S) \sin \frac{2n\pi S}{L}\, dS \qquad 4.57$$

and $$Y(S) = -h(1 + \sin \frac{\pi}{l}(S-L))\,;\ L - \frac{l}{2} \leq S \leq L$$

$$= -h(1 - \sin \frac{\pi}{l}(S-L));\ L \leq S \leq L + \frac{l}{2}$$

$$= 0 \text{ otherwise.}$$

4.2.1. (a) Calculation of Fourier Constants a_o, a_n and b_n:

$$a_o = \frac{2}{L} \int_0^L Y(S)\, dS$$

$$= \frac{2}{L} \int_{L-\frac{l}{2}}^{L} -h(1 + \sin \frac{\pi}{l}(S-L))\, dS + \frac{2}{L} \int_{L}^{L+\frac{l}{2}} -h(1 - \sin \frac{\pi}{l}(S-L))\, dS$$

$$= -2h \frac{l}{L} (1 - \frac{2}{\pi})$$

$$= -2h u (1 - \frac{2}{\pi}) \text{ where } u = \frac{l}{L}$$

$$a_o = -2h u (1 - \frac{2}{\pi}) \qquad 4.58$$

$$a_n = \frac{2}{L} \int_0^L Y(S) \cos \frac{2n\pi S}{L}\, dS$$

$$= \frac{2}{L} \int_{L-\frac{l}{2}}^{L} \{-h(1 + \sin \frac{\pi}{l}(S-L))\} \cos \frac{2n\pi S}{L}\, dS$$

$$+\frac{2}{L}\int_{L}^{L+\frac{1}{2}}\left(-h\left(1-\sin\frac{\pi}{1}(S-L)\right)\right)\cos\frac{2n\pi S}{L}\,dS$$

$$=-\frac{2h}{L}\int_{L-\frac{1}{2}}^{L}\left(\cos\frac{2n\pi S}{L}+\cos\frac{2n\pi S}{L}\sin\frac{\pi}{1}(S-L)\right)dS$$

$$-\frac{2h}{L}\int_{L}^{L+\frac{1}{2}}\left(\cos\frac{2n\pi S}{L}-\cos\frac{2n\pi S}{L}\sin\frac{\pi}{1}(S-L)\right)dS$$

$$=\frac{-2h}{L}\left(\frac{\sin\frac{2n\pi S}{L}}{2n\pi/L}-\frac{1}{2}\left(\frac{\cos\pi\left(\frac{S-L}{1}+\frac{2nS}{L}\right)}{\pi\left(\frac{1}{1}+\frac{2n}{L}\right)}+\frac{\cos\pi\left(\frac{S-L}{1}-\frac{2nS}{L}\right)}{\pi\left(\frac{1}{1}-\frac{2n}{L}\right)}\right)\right)_{L-\frac{1}{2}}^{L}$$

$$-\frac{2h}{L}\left(\frac{\sin\frac{2n\pi S}{L}}{2n\pi/L}+\frac{1}{2}\left(\frac{\cos\pi\left(\frac{S-L}{1}+\frac{2nS}{L}\right)}{\pi\left(\frac{1}{1}+\frac{2n}{L}\right)}+\frac{\cos\pi\left(\frac{S-L}{1}-\frac{2nS}{L}\right)}{\pi\left(\frac{1}{1}-\frac{2n}{L}\right)}\right)\right)_{L}^{L+\frac{1}{2}}$$

On solving, it gives

$$a_n=\frac{2h}{\pi}\left[\frac{2\mu n-\sin n\pi\mu}{(1-4n^2\mu^2)n}\right] \qquad 4.59$$

$$b_n=\frac{2}{L}\int_{0}^{L}Y(S)\sin\frac{2n\pi S}{L}\,dS$$

$$= \frac{2h}{L}\int_{L-\frac{1}{2}}^{L} (-1+\sin\frac{\pi}{1}(S-L))\sin\frac{2n\pi S}{L}\,dS$$

$$-\frac{2h}{L}\int_{L}^{L+\frac{1}{2}} (1-\sin\frac{\pi}{1}(S-L))\sin\frac{2n\pi S}{L}\,dS$$

$$= \frac{-2h}{L}\left(\frac{-\cos\frac{2n\pi S}{L}}{2n\pi/L} + \frac{1}{2}\left\{\frac{\sin\pi(\frac{S-L}{1}-\frac{2nS}{L})}{\pi(\frac{1}{1}-\frac{2n}{L})}\right.\right.$$

$$\left.\left. - \frac{\sin\pi(\frac{S-L}{1}+\frac{2nS}{L})}{\pi(\frac{1}{1}+\frac{2n}{L})}\right\}\right)_{L-\frac{1}{2}}^{L} - \frac{2h}{L}\left(\frac{-\cos\frac{2n\pi S}{L}}{2n\pi/L} - \frac{1}{2}\left\{\frac{\sin\pi(\frac{S-L}{1}-\frac{2nS}{L})}{\pi(\frac{1}{1}-\frac{2n}{L})}\right.\right.$$

$$\left.\left. - \frac{\sin\pi(\frac{S-L}{1}+\frac{2nS}{L})}{\pi(\frac{1}{1}+\frac{2n}{L})}\right\}\right)_{L}^{L+\frac{1}{2}}$$

On solving it gives

$$b_n = 0 \qquad 4.60$$

Therefore $$Y(S) = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{2n\pi S}{L} \qquad 4.61$$

where $a_o = 2hu(1 - \frac{2}{\pi})$

and $a_n = \frac{-2h}{\pi}\frac{(\sin n\pi u - 2un)}{(1 - 4n^2u^2)n}$

4.2.1.(b) Calculation of displacement, acceleration and Contact force:

Equation of motion for single degree of freedom model is

$$\ddot{Z} + 2\zeta\omega_0\dot{Z} + \omega_0^2 Z = 2\zeta\omega_0\dot{Y} + \omega_0^2 Y$$

or in space coordinates

$$Z'' + 2\zeta\frac{\omega_0}{v}Z' + \frac{(\omega_0)^2}{(v)^2}Z = 2\zeta\frac{\omega_0}{v}Y' + \frac{\omega_0^2}{v^2}$$

(i) Homogenous Solution

Homogenous solution of the left hand side is

$$x_{ih} = \beta_i e^{\frac{\alpha_i s}{v}} \quad \text{from equation 2.7.}$$

(ii) Non-homogenous solution

From equation 2.9 we have

$$x_{inh} = \int_0^s h_i(s-p)\frac{\Phi_i}{M_i}\left(2\zeta\omega_0\frac{dy}{dp}\frac{dp}{dt} + \omega_0^2 Y(p)\right)dp$$

$$\text{where } Y(p) = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{2n\pi p}{L}$$

$$Y'(p) = \frac{dy(p)}{dp} = \frac{2\pi}{L}\sum_{n=1}^{\infty} -n\, a_n \sin\frac{2n\pi p}{L}$$

$$h_i(s-p) = \frac{e^{\frac{\alpha_i}{v}(s-p)}}{v} \qquad i = 1,2$$

substituting, we get

$$x_{inh} = \int_0^s \frac{e^{\frac{\alpha_i}{v}(s-p)}}{v}\frac{\Phi_i}{M_i}\left\{2\zeta\omega_0 v\left(\frac{2\pi}{L}\sum_{n=1}^{\infty} -n\, a_n \sin\frac{2n\pi p}{L}\right) + \omega_0^2\left(\frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{2n\pi p}{L}\right)\right\}dp$$

$$= \frac{\Phi_i}{M_i v} e^{\frac{\alpha_i s}{v}}\left(\frac{a_o}{2}\omega_0^2 I_1 + \sum_{n=1}^{\infty}\left\{-4\zeta\frac{\omega_0 v\pi n}{L} I_2 + \omega_0^2 I_3\right\} a_n\right) \qquad 4.62$$

where

$$I_1 = \frac{v}{\alpha_i}\left(1 - e^{\frac{-\alpha_i S}{v}}\right) \qquad 4.63$$

$$I_2 = \frac{1}{\left(\left(\frac{-\alpha_i}{v}\right)^2 + \left(\frac{2n\pi}{L}\right)^2\right)^{\frac{1}{2}}}\left(e^{\frac{\alpha_i S}{v}}\sin\left(\frac{2n\pi S}{L} - \gamma_i\right) + \sin\gamma_i\right) \qquad 4.64$$

$$I_3 = \left\{\frac{1}{\left(\frac{-\alpha_i}{v}\right)^2 + \left(\frac{2n\pi}{L}\right)^2}\right\}^{\frac{1}{2}}\left(e^{\frac{-\alpha_i S}{v}}\cos\left(\frac{2n\pi S}{L} - \gamma_i\right) + \cos(\gamma_i)\right) \qquad 4.65$$

$$\gamma_i = \tan^{-1}\frac{2n\pi v}{-L\alpha_i}$$

Therefore

i) Absolute Displacement: $Z(S)$:

$$Z(S) = \sum_{i=1}^{2} x_{ih}\,\phi_i + \sum_{i=1}^{2} x_{inh}\,\phi_i$$

$$= \sum_{i=1}^{2}\phi_i\,\beta_i\,e^{\frac{\alpha_i s}{v}} + \sum_{i=1}^{2}\frac{\phi_i^2\,e^{\frac{\alpha_i s}{v}}}{M_i V}\left\{\frac{a_o}{2}\,\omega_0^2 I_1\right.$$

$$\left. + \sum_{n=1}^{\infty}\left(-4\zeta\omega_0 v\,\frac{\pi n}{L}\,I_2 + \omega_0^2 I_3\right)a_n\right\}$$

ii) Absolute Acceleration: $\ddot{Z}(S)$

$$\ddot{Z}(S) = \sum_{i=1}^{2}\phi_i\,\frac{\alpha_i^2}{}\,\beta_i e^{\frac{\alpha_i S}{v}} + \sum_{i=1}^{2}\frac{\phi_i^2}{M_i V}$$

$$\left(\frac{a_o}{2} \omega_0^2 \alpha_i v \; e^{\frac{\alpha_i S}{V}} + \sum_{n=1}^{\infty} \left(-4\zeta\omega_0 v \frac{\pi n}{L} I_4 + \omega_0^2 I_5 \right) a_n \right.$$

where

$$I_4 = \frac{v^2}{\left\{ \left(-\frac{\alpha_i}{v} \right)^2 + \left(\frac{2n\pi}{L} \right)^2 \right\}^{\frac{1}{2}}} \left\{ - \frac{(2n\pi)^2}{(L)^2} \sin\left(\frac{2n\pi S}{L} - \gamma_i \right) + \frac{(\alpha_i)^2}{(v)^2} e^{\frac{\alpha_i S}{v}} \sin \gamma_i \right\}$$

$$I_5 = \frac{v^2}{\left(\left(\frac{-\alpha_i}{v} \right)^2 + \left(\frac{2n\pi}{L} \right)^2 \right)^{\frac{1}{2}}} \left\{ - \frac{(2n\pi)^2}{(L)^2} \cos\left(\frac{2n\pi S}{L} - \gamma_i \right) + \frac{(\alpha_i)^2}{(v)^2} e^{\frac{\alpha_i S}{v}} \cos \gamma_i \right\}$$

iii) <u>Contact Force</u> (F(S))

$$F(S) = M\ddot{Z}(S) + m\ddot{Y}(S) + (M+m)g$$

$$= M \left\{ \sum_{i=1}^{2} \Phi_i \alpha_i^2 \beta_i e^{\frac{\alpha_i S}{v}} + \sum_{i=1}^{2} \frac{\Phi_i^2}{M_i v} \left(\frac{a_o}{2} \omega_0^2 \alpha_i v \right. \right.$$

$$\left. e^{\frac{\alpha_i S}{v}} + \sum_{n=1}^{2} \left(- 4\zeta\omega_0 v \frac{\pi n}{L} I_4 + \omega_0^2 I_5 \right) a_n \right\}$$

$$- \left(\frac{2 V \pi}{L} \right)^2 \sum_{n=1}^{\infty} n^2 a_n \cos \frac{2n\pi S}{L} + (M+m)g$$

4.2.2. <u>Results</u>

Following observations may be made (Fig. 4.30 to 4.32).

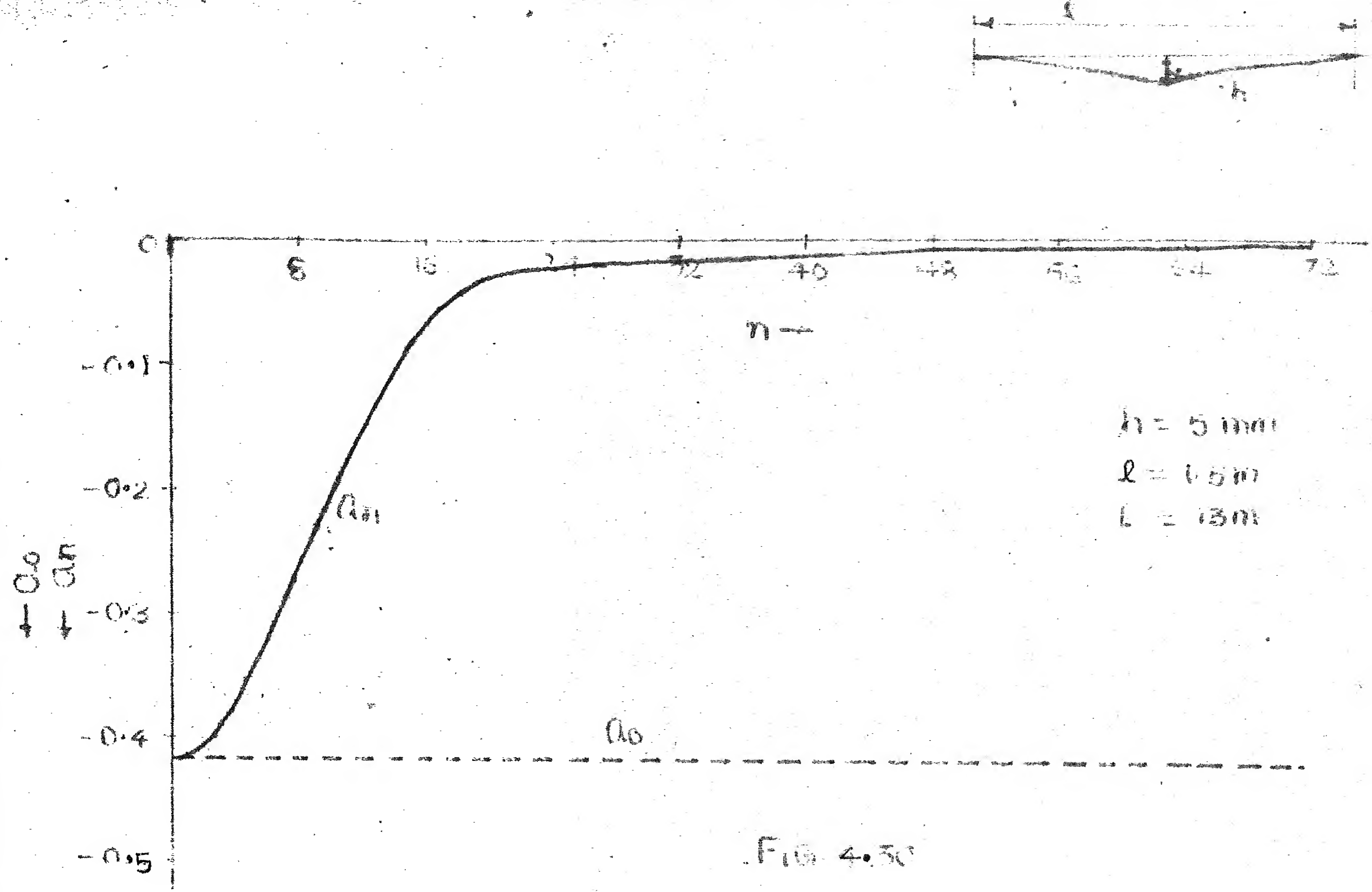
1. In Fig. 4.30, Fourier constants a_o and a_n have been plotted against n. Fixed parameters are l = 1.5m., L = 13m and h = 5mm. It is seen that as n increases value

of a_n decreases very sharply at upto n = 20, and then decreases slowly. Value of an a_n at n = 1 is about 175 times value of a_n at n = 70.

2. Shape of the pulse is obtained by two methods. First by directly taking assumed shape and secondly by considering equivalent Fourier Series. A comparison between forms of two pulse shape obtained by two methods shows that two method gives identical results.

3. On Fig. 4.32, ratio of absolute displacement to the h has been obtained by two methods as in case of pulse shape. Here also, it is seen that the two method gives identical results.

Thus it is concluded that response values obtained by Fourier analysis are identical to that given by single pulse excitation analysis.



0
-0.1
-0.2
-0.3
-0.4
-0.5
8
16
24
32
40
48
56
64
72
n →
a₀ ↓ aₙ ↓
aₙ
a₀
h = 5 mm
l = 1.5 m
L = 13 m
l
h

FIG. 4.30

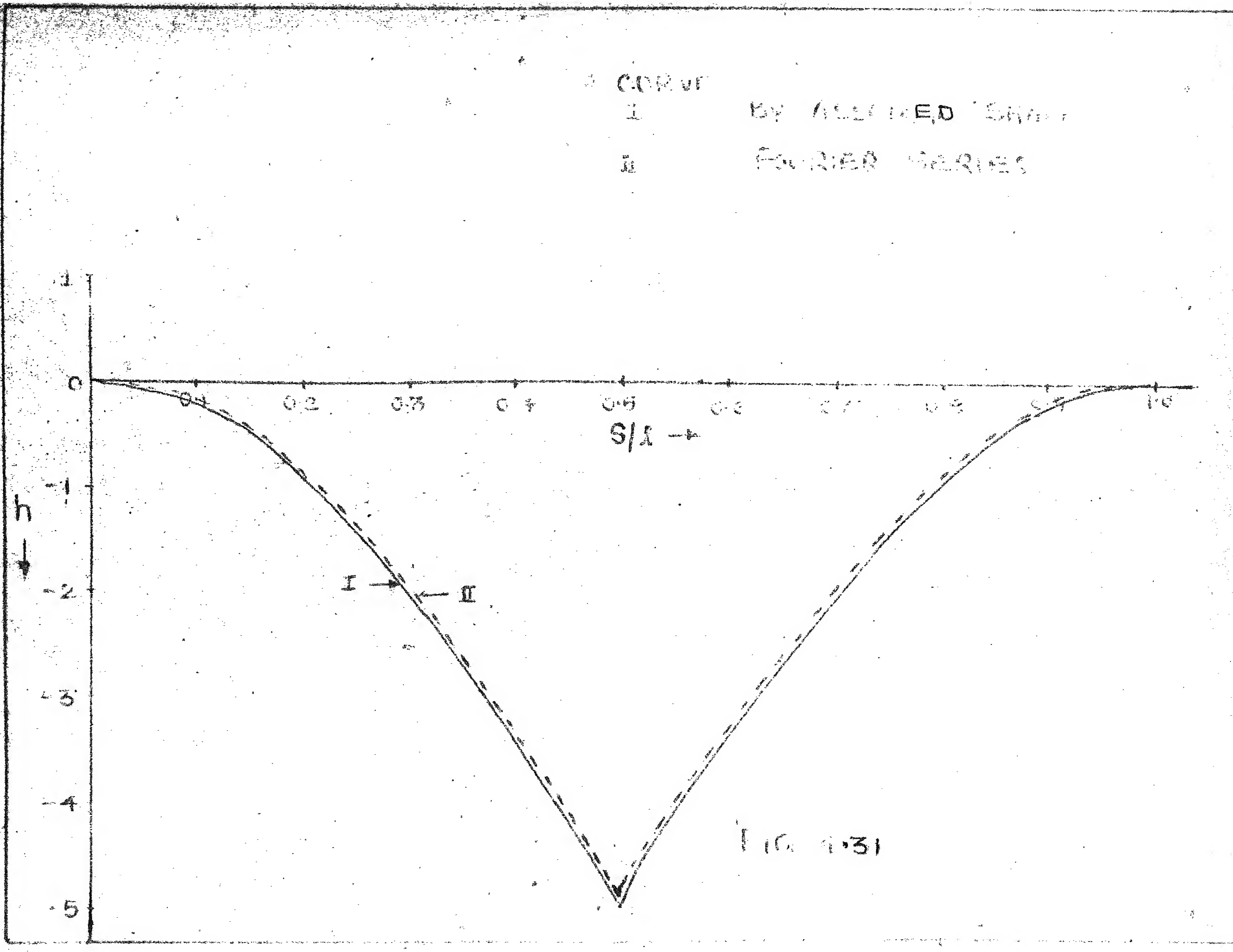


FIG. 4.31

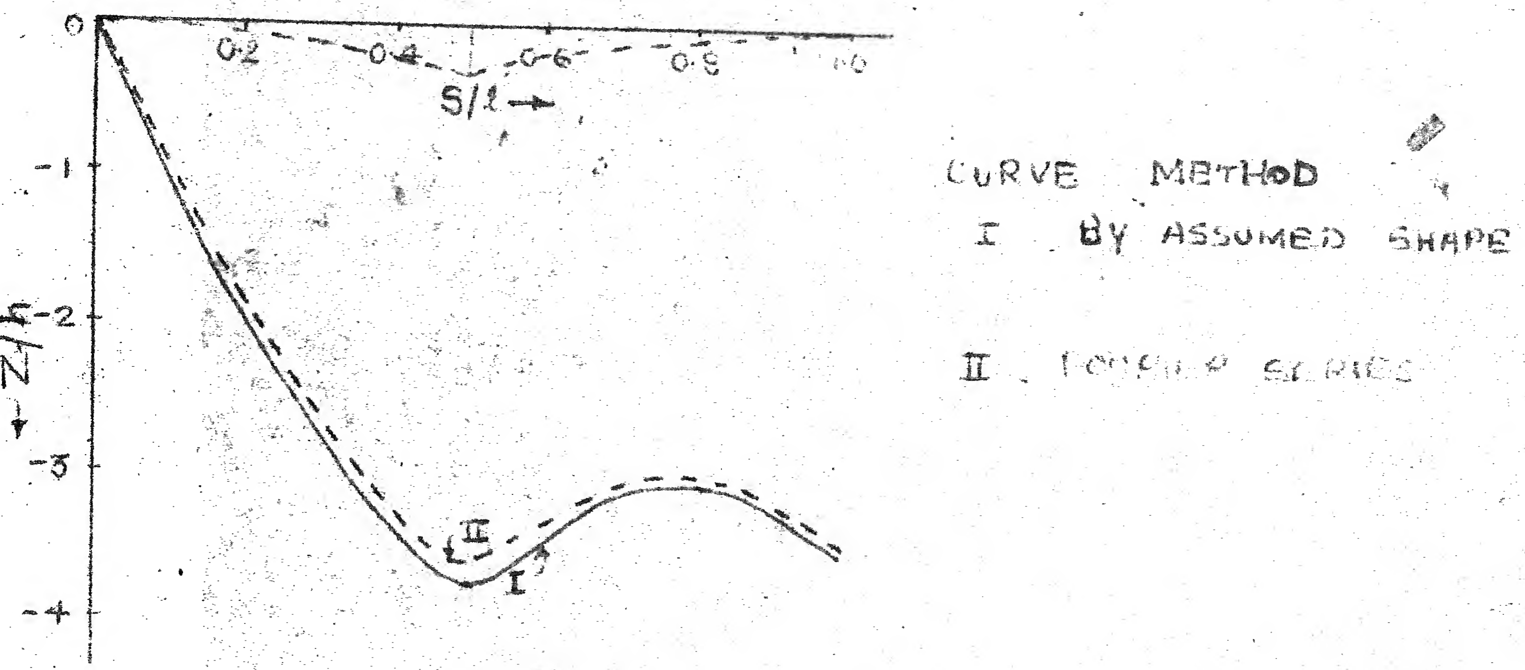

0
0.2
0.4
0.6
0.8
1.0
S/l →
-1
-2
-3
-4
Z/h →
II
I
CURVE METHOD
I BY ASSUMED SHAPE
II FOURIER SERIES

FIG. 4.32

CHAPTER 5

RESPONSE OF PERIODICALLY OCCURRING RANDOM LOW JOINTS

INTRODUCTION

In the Chapter 4, we have calculated all response parameters deterministically by taking known of values of l and h. In this chapter., second order statistics of absolute displacement, acceleration and contact force are calculated by considering length l and depth h of the pulse as random. Such a analysis is useful in calculating the fatigue life of the structural components. The occurrances of the pulses are not random. ~~Still~~ and they occur periodically. Following assumptions are made.

i) Length and depth are identically distributed and completely correlated random variables. They are related by the relation h = cl where c is a constant. This linear relation seems to be practicable as it is seen that increase in l, increases depth h also (Fig. 5.2).

ii) l is uniformly distributed between l_1 and l_2 as shown in fig. 5.1.

Probability density function of l is given by

$$P(l) = \frac{1}{l_2 - l_1}; \; l_1 \leq l \leq l_2$$

$$= 0 \quad \text{otherwise.}$$

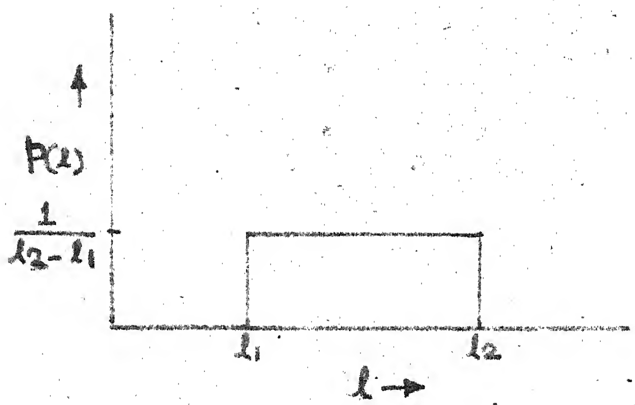


FIG 5.1

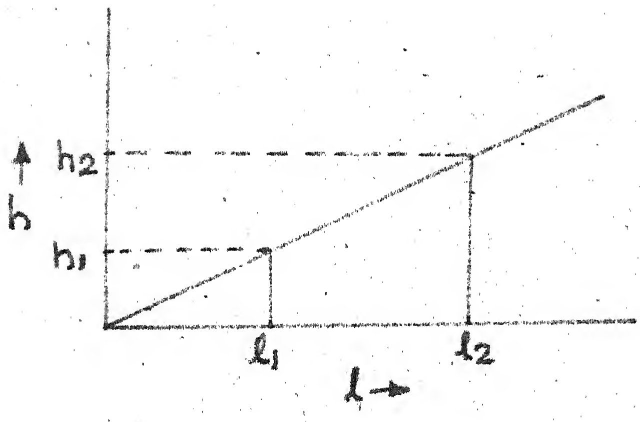


FIG 5.2

$$\left(\frac{\omega_0^2}{u}\left(1 - e^{u(S-nL+\frac{1}{2})}\right) + \frac{T}{A}\left(u\sin(\Omega(S-nL)+\gamma) + \Omega\cos(\Omega(S-nL)+\gamma)\right) + \frac{T}{A}e^{u(S-nL+\frac{1}{2})}\left(u\cos\gamma - \Omega\sin\gamma\right)\right)$$

WHERE $cl = h$ 5.2

Expected value of a function f(x) is defined as

$$E(f(x)) = \int p_x(x)f(x)dx \qquad 5.3$$

where x is a random variable and

$p_x(x)$ is the probability density function

$$E(f^2(x)) = \int p_x(x)f^2(x)\, dx \qquad 5.4$$

and variance of f(x)

$$= \sigma^2_{f(x)} = E(f^2(x)) - \left(E\ f(x)\right)^2 \qquad 5.5$$

using above definitions, expected value and variance are calculated.

i) <u>Expected value of</u> $Z_{n,1}(S)$: $E(Z_{n,1}(S))$

$$E\left\{Z_{n,1}(S)\right\} = \int Z_{n,1}(S)\, p_{(l)}dl$$

Where $p(l)$ is the probability density function hence

$$E\left\{Z_{n,1}(S)\right\} = \int_{l_1}^{l_2} Z_{n,1}(S)\frac{1}{l_2 - l_1}\, dl$$

$$= \frac{1}{l_2 - l_1}\int_{l_1}^{l_2} Z_{n,1}(S)\, dl \qquad 5.6$$

A closed form solution of the above integral is not possible as $Z_{n,1}(S)$ is not a simple function l. This integral is evaluated numarically.

(ii) Variance of $Z_{n,1}(S)$: $\sigma^2 Z_{n,1}(S)$

$$E(Z^2_{n,1}(S)) = \int_{l_1}^{l_2} Z^2_{n,1}(S) \frac{1}{l_2 - l_1} \, dl$$

$$= \frac{1}{l_2 - l_1} \int_{l_1}^{l_2} Z^2_{n,1}(S) \, dl \qquad 5.7$$

Hence,

$$\sigma^2 Z_{n,1}(S) = E\left(Z^2_{n,1}(S)\right) - \left(E(Z_{n,1}(S))\right)^2$$

$$= \frac{1}{l_2 - l_1} \int_{l_1}^{l_2} Z^2_{n,1}(S) \, dl - \left(\frac{1}{l_2 - l_1} \int_{l_1}^{l_2} Z_{n,1}(S) \, dl\right)^2 \qquad 5.8$$

2) <u>Absolute Acceleration</u>: $\ddot{Z}_{n,1}(S)$

$$\ddot{Z}_{n,1}(S) = \sum_{i=1}^{2} \Phi_i \beta_i \alpha_i^2 \, e^{\frac{\alpha_i}{v}(S-nL)+\frac{1}{2})} + \sum_{i=1}^{2}$$

$$\frac{C1 \, \Phi_i^2}{M_i} \, v\left(-u\omega_0^2 e^{u(S-nL+\frac{1}{2})} + \frac{T}{A}\Omega^2 \left(-u\sin(\Omega(S-nL)+\gamma)\right.\right.$$

$$\left. -\Omega\cos(\Omega(S-nL)+\gamma)\right) + \frac{T}{A}u^2 e^{u((S-nL)+\frac{1}{2})}\left(u\cos\gamma\right.$$

$$\left.\left. -\Omega\sin\gamma\right)\right) \qquad 5.9$$

i) Expected Value of $\ddot{Z}_{n,1}(S)$: $E(\ddot{Z}_{n,1}(S))$

$$E(\ddot{Z}_{n,1}(S)) = \frac{1}{l_2 - l_1} \int_{l_1}^{l_2} \ddot{Z}_{n,1}(S) \, dl \qquad 5.10$$

ii) Variance of $\ddot{Z}_{n,1}(S)$: $\sigma^2 \ddot{Z}_{n,1}(S)$

$$E\left(\ddot{Z}^2_{n,1}(S)\right) = \frac{1}{l_2 - l_1} \int_{l_1}^{l_2} \ddot{Z}^2_{n,1} \, dl \qquad 5.11$$

hence

$$\sigma^2_{\ddot{Z}_{n,1}(S)} = E\left(\ddot{Z}^2_{n,2}(S)\right) - \left\{ E\left(\ddot{Z}_{n,1}(S)\right) \right\}^2$$

$$= \frac{1}{l_2 - l_1} \int_{l_1}^{l_2} \ddot{Z}^2_{n,1}(S) \, dl - \left(\frac{1}{l_2 - l_1} \int_{l_1}^{l_2} \ddot{Z}_{n,1}(S) \, dl \right)^2 \qquad 5.12$$

3. <u>Contact Force:</u> $F_{n,1}(S)$

$$F_{n,1}(S) = M \ddot{Z}_{n,1}(S) + m \ddot{y}_n(S) + (M+m) g \qquad 5.13$$

i) Expected value of $F_{n,1}(S)$: $E(F_{n,1}(S)$

$$E(F_{n,1}(S)) = E(M \ddot{Z}_{n,1}(S)) + E(m \ddot{y}_n(S)) + (M+M) g$$

$$= ME(\ddot{Z}_{n,1}(S)) + ME(\ddot{y}_n(S)) + (M+m) g$$

where $E(\ddot{Z}_{n,1}(S)) = \frac{1}{l_2 - l_1} \int_{l_1}^{l_2} \ddot{Z}_{n,1}(S) \, dl$

hence

$$E(F_{n,1}(S)) = M \frac{1}{l_2 - l_1} \int_{l_1}^{l_2} \ddot{Z}_{n,1}(S) \, dl + m \frac{1}{l_2 - l_1} \int_{l_1}^{l_2} \ddot{y}_n(S) \, dl + (M+m) g \qquad 5.14$$

ii) Variance of $F_{n,1}(S)$: $\sigma^2_{F_{n,1}(S)}$

$$E(F^2_{n,1}(S)) = \frac{1}{l_2 - l_1} \int_{l_1}^{l_2} F^2_{n,1}(S)\,dl$$

$$= \frac{1}{l_2 - l_1} \int_{l_1}^{l_2} \left(M\ddot{Z}_{n,1}(S) + m\ddot{y}_n(S) + (M+m)g \right)^2 dl \qquad 5.15$$

hence

$$\sigma^2_{F_{n,1}(S)} = E(F^2_{n,1}(S)) - \left\{ E(F_{n,1}(S)) \right\}^2$$

$$= \frac{1}{l_2 - l_1} \int_{l_1}^{l_2} \left(M\ddot{Z}_{n,1}(S) + m\ddot{y}_n(S) + (M+m)g \right)^2 dl$$

$$- \left(\frac{1}{l_2 - l_1} \int_{l_1}^{l_2} \left(M\ddot{Z}_{n,1}(S) + m\ddot{y}_n(S) + (M+m)g \right) dl \right)^2 \qquad 5.16$$

(b) <u>Second Region</u>: $nL \leq S \leq nL + \frac{1}{2}$

1) <u>Absolute Displacement</u>: $Z_{n,2}(S)$

$$Z_{n,2}(S) = \sum_{i=1}^{2} \beta_i \Phi_i e^{\frac{\alpha_i}{v}(S-nL)} + \sum_{i=1}^{2} \frac{Cl\,\Phi_i^2}{M_i\,v} \left\{ \frac{\omega_0^2}{u} \left(1 - e^{\frac{ul}{2}} \right) + \frac{T}{A} \left\{ \Omega\cos\gamma + u\sin\gamma \right\} + \frac{T}{A} e^{\frac{ul}{2}} \left\{ u\cos\gamma - \Omega\sin\gamma \right\} \right.$$

$$\left. + \frac{\omega_0^2}{u} \left(1 - e^{u(S-nL)}\right) - \frac{T}{A} \left\{ u\sin(\Omega(S-nL)+\gamma) + \Omega\cos(\Omega(S-nL) + \gamma) \right\} + \frac{T}{A} e^{u(S-nL)} \left\{ u\sin\gamma + \Omega\cos\gamma \right\} \right\} \qquad 5.17$$

i) Expected Value of $Z_{n,2}(S)$: $E(Z_{n,2}(S))$

$$E(Z_{n,2}(S)) = \frac{1}{l_2 - l_1} \int_{l_1}^{l_2} Z_{n,2}(S)\,dl \qquad 5.18$$

ii) Variance of $Z_{n,2}(S)$: $\sigma^2_{Z_{n,2}(S)}$

$$E(Z^2_{n,2}(S)) = \frac{1}{l_2 - l_1} \int_{l_1}^{l_2} Z^2_{n,2}(S)\, dl \qquad 5.18$$

hence

$$\sigma^2_{Z_{n,2}(S)} = E\left(Z^2_{n,2}(S)\right) - \left\{E\left(Z_{n,2}(S)\right)\right\}^2$$

$$= \frac{1}{l_2 - l_1} \int_{l_1}^{l_2} Z^2_{n,2}(S)\, dl - \left(\frac{1}{l_2 - l_1} \int_{l_1}^{l_2} Z_{n,2}(S)\, dl \right)^2 \qquad 5.20$$

2) Absolute Acceleration: $\ddot{Z}_{n,2}(S)$

$$Z_{n,2}(S) = \sum_{i=1}^{2} \phi_i\, \beta_i\, \alpha_i^2\, e^{\frac{\alpha_i}{v}(S-nL)}$$

$$+ \sum_{i=1}^{2} \frac{cl\, \phi_i^2}{M_i} v \left(-u\omega_0^2\, e^{u(S-nL)} \right.$$

$$- \frac{T}{A}\Omega^2 \left(-u\sin(\Omega(S-nL)+\gamma) - \Omega\cos(\Omega(S-nL)+\gamma)\right) + \frac{T}{A}u^2$$

$$\left. e^{u(S-nL)} (u\sin\gamma + \Omega\cos\gamma) \right) \qquad 5.21$$

i) Expected value of $\ddot{Z}_{n,2}(S)$: $E(\ddot{Z}_{n,2}(S))$

$$E(\ddot{Z}_{n,2}(S)) = \frac{1}{l_2 - l_1} \int_{l_1}^{l_2} \ddot{Z}_{n,2}(S)\, dl \qquad 5.22$$

ii) Variance of $\ddot{Z}_{n,2}(S)$: $\sigma^2_{\ddot{Z}_{n,2}(S)}$

$$E(\ddot{Z}^2_{n,2}(S)) = \frac{1}{l_2 - l_1} \int_{l_1}^{l_2} \ddot{Z}^2_{n,2}(S)\, dl \qquad 5.23$$

hence

$$\sigma^2_{\ddot{Z}_{n,2(S)}} = E\{\ddot{Z}^2_{n,2}(S)\} - \{E(\ddot{Z}_{n,2(S)})\}^2$$

$$= \frac{1}{l_2 - l_1}\int_{l_1}^{l_2} \ddot{Z}^2_{n,2}(S)\, dl - \left(\frac{1}{l_2 - l_1}\int_{l_1}^{l_2} \ddot{Z}_{n,2(S)}\, dl\right)^2 \qquad 5.24$$

3. <u>Contact Force:</u> $F_{n,2(S)}$

$$F_{n,2(S)} = M\ddot{Z}_{n,2(S)} + m\ddot{Y}n(S) + (M+m)g \qquad 5.25$$

i) Expected value of $F_{n,2}(S)$: $E(F_{n,2(S)})$

$$E(F_{n,2(S)}) = E(M\ddot{Z}_{n,2(S)} + m\ddot{Y}_{n(S)} + (M+m)g)$$

$$= ME(\ddot{Z}_{n,2(S)}) + mE(\ddot{Y}_{n(S)}) + (M+m)g$$

where $E(\ddot{Z}_{n,2(S)}) = \frac{1}{l_2 - l_1}\int_{l_1}^{l_2} \ddot{Z}_{n,2(S)}\, dl$

$$E(\ddot{Y}_n(S)) = \frac{1}{l_2 - l_1}\int_{l_1}^{l_2} \ddot{Y}_n(S)\, dl$$

$$E(\ddot{Y}_n(S)) = \frac{1}{l_2 - l_1}\int_{l_1}^{l_2} \ddot{Y}_n(S)\, dl$$

hence

$$E(F_{n,2(S)}) = M\frac{1}{l_2 - l_1}\int_{l_1}^{l_2} \ddot{Z}_{n,2(S)}\, dl + m\frac{1}{l_2 - l_1}\int_{l_1}^{l_2} \ddot{Y}_n(S)\, dl + (M+m)g \qquad 5.26$$

ii) Variance of $F_{n,2}(S)$: $\sigma^2 F_{n,2(S)}$

$$E\left(F^2_{n,2(S)}\right) = \frac{1}{l_2 - l_1}\int_{l_1}^{l_2} F^2_{n,2(S)}\,dl$$

$$= \frac{1}{l_2 - l_1}\int_{l_1}^{l_2} \left(M\ddot{Z}_{n,2(S)} + m\ddot{Y}_{n(S)} + (M+m)g\right)^2 dl \tag{5.27}$$

hence,

$$\sigma^2 F_{n,2(S)} = E\left(F^2_{n,2}(S)\right) - \left\{E\left(F_{n,2(S)}\right)\right\}^2$$

$$= \left(\frac{1}{l_2 - l_1}\int_{l_1}^{l_2}\left((M\ddot{Z}_{n,2(S)} + m\ddot{Y}_{n(S)} + (M+m)g)\right)^2 dl\right) - \left\{\frac{1}{l_2-l_1}\int_{l_1}^{l_2}(M\ddot{Z}_{n,2}(S) + m\ddot{Y}n(S) + (M+m)g)\,dl\right\}^2 \tag{5.28}$$

c) Third Region: $S \nless nL + \frac{1}{2}$

1) Absolute Displacement: $Z_{n,3(S)}$

$$Z_{n,3(S)} = Z_{sn}g(S-S_n) + \dot{Z}_{Sn}\,h(S-S_n) \tag{5.29}$$

Where Z_{Sn}, $g(S-S_n)$, $\dot{Z}_{Sn}$ and $h(S-S_n)$ are as defined in equations 4.25 4,26, 4.27 and 4.28.

i) Expected Value of $Z_{n,3(S)}$: $E(Z_{n,3(S)})$

$$E\left(Z_{n,3(S)}\right) = \frac{1}{l_2 - l_1}\int_{l_1}^{l_2} Z_{n,3(S)}\,dl \tag{5.30}$$

ii) Variance of $Z_{n,3(S)}$: $\sigma^2 Z_{n,3(S)}\,dl$ 5.31

$$\text{hence } \sigma^2 Z_{n,3(S)} = E\left(Z^2_{n,3(S)}\right) - \left(E(Z_{n,3(S)})\right)^2$$

$$= \frac{1}{l_2-l_1}\int_{l_1}^{l_2} Z^2_{n,3(S)}\,dl - \left(\frac{1}{l_2-l_1}\int_{l_1}^{l_2} Z_{n,3(S)}\,dl\right)^2 \tag{5.32}$$

2) <u>Absolute Acceleration</u>: $\ddot{Z}_{n,3(S)}$

$$\ddot{Z}_{n,3(S)} = Z_{Sn}\ddot{g}(S-S_n) + \dot{Z}_{Sn}\ddot{h}(S-S_n) \qquad 5.33$$

Where

$$\ddot{g}(S-S_n) = e^{-\frac{\zeta\omega_0}{v}(S-S_n)} \left(\frac{\zeta}{(1-\zeta^2)^{\frac{1}{2}}}(-2\zeta\omega_0\omega_d) + (\zeta^2\omega_0^2 - \omega_d^2) \right)$$

$$\sin\frac{\omega_d}{v}(S-S_n) + \left(\frac{-\zeta}{(1-\zeta^2)^{\frac{1}{2}}}(-2\zeta\omega_0\omega_d) + (\zeta^2\omega_0^2 - \omega_d^2) \right)$$

$$\left.\cos\frac{\omega_d}{v}(S-S_n)\right)$$

$$\ddot{h}(S-S_n) = \frac{e^{-\frac{\zeta\omega_0}{v}(S-S_n)}}{\omega_d} \left(\sin\frac{\omega_d}{v}(S-S_n)\ (\zeta^2\omega_0^2\ \ \omega_d^2) \right.$$

$$\left. -\ 2\zeta\omega_0\omega_d\cos\frac{\omega_d}{v}(S-S_n) \right)$$

i) <u>Expected value of</u> $\ddot{Z}_{n,3(S)}$: $E(\ddot{Z}_{n,3}(S))$

$$E(\ddot{Z}_{n,3(S)}) = \frac{1}{l_2-l_1}\int_{l_1}^{l_2} \ddot{Z}_{n,3(S)}\,dl \qquad 5.34$$

ii) <u>Variance of</u> $\ddot{Z}_{n,3(S)}$: $\sigma\ddot{Z}^2_{n,3(S)}$

$$E(\ddot{Z}^2_{n,3(S)}) = \frac{1}{l_2-l_1}\int_{l_1}^{l_2} \ddot{Z}^2_{n,3}(S)\,dl \qquad 5.35$$

hence

$$\sigma\ddot{Z}^2_{n,3(S)} = E(\ddot{Z}^2_{n,3}(S)) - \left(E(\ddot{Z}_{n,3}(S)) \right)^2$$

$$= \frac{1}{l_2-l_1}\int_{l_1}^{l_2} \ddot{Z}^2_{n,3}(S)\,dl - \left(\frac{1}{l_2-l_1}\int_{l_1}^{l_2} \ddot{Z}_{n,3(S)}\,dl \right)^2 \qquad 5.36$$

3) <u>Contact Force</u>: $F_{n,3(S)}$ 5.36

$$F_{n,3(S)} = M\ddot{Z}_{n,3(S)} + (M+m)g \qquad 5.37$$

i) Expected value of $F_{n,3}(S)$: $E(F_{n,3(S)})$

$$E(F_{n,3(S)}) = ME\,(\ddot{Z}_{n,3(S)} + (M+m)g$$

$$= M\frac{1}{l_2 - l_1}\int_{l_1}^{l_2} \ddot{Z}_{n,3(S)}\,dl + (M+m)g \qquad 5.38$$

ii) <u>Variance of</u> $F_{n,3(S)}$: $F_{n,3(S)}$

$$E(F^2_{n,3(S)}) = \frac{1}{l_2 - l_1}\int_{l_1}^{l_2} F^2_{n,3(S)}\,dl$$

$$= \frac{1}{l_2 - l_1}\int_{l_1}^{l_2} (M\ddot{Z}_{n,3(S)} + (M+m)g\,)^2 dl \qquad 5.39$$

hence $\sigma^2 F_{n,3(S)} = E(F^2_{n,3(S)}) - (E(F_{n,3(S)})^2$

$$= \frac{1}{l_2 - l_1}\int_{l_1}^{l_2} (M\ddot{Z}_{n,3(S)} + (M*m)g)^2 dl$$

$$-\left(\frac{1}{l_2 - l_1}\int_{l_1}^{l_2} M\ddot{Z}_{n,3(S)} + (M+m)g\,dl\right)^2 \qquad 5.40$$

THESE ARE THE SECOND order statistics of response parameters due to n_{th} pulse only. Second order statistics for series of N pulses are obtained by Superimposing second order statics of response parameters due to each pulse. Here also there will be three regions.

(A) First Region: $(N+1)L - \frac{1}{2} \leq S \leq (N+1)L$

(1) Absolute Displacement: $Z(S)$

$$Z(S) = \sum_{n=1}^{N} Z_{n,3}(S) + Z_{N+1,1}(S) \qquad 5.41$$

(i) Expected value of $Z(S)$: $E(Z(S))$

$$E(Z(S) = \sum_{n=1}^{N} E(Z_{n,3}(S) + Z_{N+1,1}(S)) \qquad 5.42$$

(ii) Variance of $Z(S)$: $\sigma^2_Z(S)$

$$E(Z^2(S)) = \sum_{n=1}^{N} E(Z^2_{n,3}(S)) + E(Z^2_{N+1}(S)) \qquad 5.43$$

as $E(Z_{n,3}(S)\ Z_{N+1}(S))$ is zero

$$\text{hence } \sigma^2_{Z(S)} = E(Z^2(S)) - \{E(Z(S))\}^2 \qquad 5.44$$

(2) Absolute Acceleration: $\ddot{Z}(S)$

$$\ddot{Z}(S) = \sum_{n=1}^{N} \ddot{Z}_{n,3}(S) + \ddot{Z}_{N+1,1}(S) \qquad 5.45$$

(i) Expected Value of $\ddot{Z}(S)$: $E(\ddot{Z}(S))$

$$E(\ddot{Z}(S)) = \sum_{n=1}^{N} E(\ddot{Z}(S)) + E(\ddot{Z}_{N+1,1}(S)) \qquad 5.46$$

(ii) Variance of $\ddot{Z}(S)$: $\sigma_{\ddot{Z}}^{2}(S)$

$$E(\ddot{Z}^2(S)) = \sum_{n=1}^{N} E(\ddot{Z}^2_{n,3}(S)) + E(\ddot{Z}^2_{N+1,1}(S)) \qquad 5.47$$

as $E(\ddot{Z}_{n,3}(S)\ \ddot{Z}_{N+1}(S))$ is zero

as $E(\ddot{Z}_{n,3}(S)\ \ddot{Z}_{N+1}(S))$ is zero

hence $\sigma^2_{\ddot{Z}(S)} = E(\ddot{Z}^2(S)) - (E(\ddot{Z}(S)))^2$ 5.48

3) <u>Contact Force</u>: $F(S)$

$F(S) = M\ddot{Z}(S) + (M+m)g + m\ddot{Y}_n(S)$ 5.49

i) <u>Expected value</u> of $F(S)$: $E(F(S))$

$E(F(S)) = ME(\ddot{Z}(S)) + mE(\ddot{Y}_n(S)) + (M+m)g$ 5.50

ii) <u>Variance of</u> $F(S)$: $\sigma^2 F(S)$

$E(F^2(S)) = M^2E(\ddot{Z}^2(S)) + m^2E(\ddot{Y}_n^2(S)) + (M+m)g)^2$

$+2MmE(\ddot{Z}(S)\ \ddot{Y}_n(S)) + 2M(M+m)g\ E$

$(\ddot{Z}(S)) + 2m(M+m)E(\ddot{Y}_n(S))g$ 5.51

hence $\sigma^2_F(S) = E(F^2(S)) - (E(F(S)))^2$ 5.52

(B) SECOND REGION: $(N+1)L \leq S \leq (N+1)L + \frac{1}{2}$

(1) Absolute Displacement: $Z(S)$

$Z(S) = \sum_{n=1}^{N} Z_{n,3}(S) + Z_{N+1,2}(S)$ 5.53

(i) Expected Value of $Z(S)$: $E(Z(S)$

$E(Z(S) = \sum_{n=1}^{N} E(Z_{n,3}(S)) + E(Z_{N+1,2}(S))$ 5.54

(ii) Variance of $Z(S)$ $\sigma^2_{Z(S)}$

$E(Z^2(S)) = \sum_{n=1}^{N} E(Z^2_{n,3}(S)) + E(Z^2_{N+1,2}(S))$ 5.55

as $E(Z_{n,3(S)} Z_{N+1,2(S)})$ is zero,

hence $\sigma^2_{Z(S)} = E(Z^2(S)) - (E(Z(S)))^2$ 5.56

(2) Absolute Acceleration: $\ddot{Z}(S)$

$$\ddot{Z}(S) = \sum_{n=1}^{N} \ddot{Z}_{n,3(S)} + \ddot{Z}_{N+1,2(S)} \qquad 5.57$$

I) Expected value of $\ddot{Z}(S)$: $E(\ddot{Z}(S))$

$$E(\ddot{Z}(S)) = \sum_{n=1}^{N} E(\ddot{Z}_{n,3}(S)) + E(\ddot{Z}_{N+1,2}(S)) \qquad 5.58$$

ii) Variance of $\ddot{Z}(S)$: $\sigma^2_{\ddot{Z}}(S)$

$$E(\ddot{Z}^2(S)) = \sum_{n=1}^{N} E(\ddot{Z}^2_{n,3(S)}) + E(\ddot{Z}^2_{N+1,2}(S)) \qquad 5.59$$

as $E(\ddot{Z}_{n,3(S)} \ddot{Z}_{N+1,2(S)})$ is zero.

hence $\sigma^2 \ddot{Z}(S) = E(\ddot{Z}^2(S)) - (E(\ddot{Z}(S)))^2$ 5.60

(3) <u>Contact Force:</u> $F(S)$

$$F(S) = M\ddot{Z}(S) + m\ddot{Y}_n(S) + (M+m)g \qquad 5.61$$

i) Expected value of $F(S)$: $E(F(S))$

$$E(F(S) = ME(\ddot{Z}(S) + mE(\ddot{Y}_n(S) + (M+m)g \qquad 5.62$$

ii) Variance of $F(S)$: $\sigma_F^2(S)$

$$E(F^2(S)) = M^2E(\ddot{Z}^2(S)) + m^2E(\ddot{Y}_n^2(S)) + (M+m)g)^2$$

$$+2MmE(\ddot{Z}(S)\ddot{Y}_n(S)) + 2M(M+m)g\,E(\ddot{Z}(S)) + 2m(M+m)E(\ddot{Y}_n(S)) \qquad 5.63$$

hence $\sigma_F^2(S) = E(F^2(S)) - ((E(F(S)))^2$ 5.64

(C) THIRD REGION: $S \geq (N+1)L + \frac{1}{2}$

1) Absolute Displacement: $Z(S)$

$$Z(S) = \sum_{n=1}^{N+1} Z_{n,3}(S) \qquad 5.65$$

i) Expected value of $Z(S)$: $E(Z(S))$

$$E(Z(S)) = \sum_{n=1}^{N+1} E(Z_{n,3}(S)) \qquad 5.66$$

ii) Variance of $Z(S)$: $\sigma^2 Z(S)$

$$E(Z^2(S)) = \sum_{n=1}^{N+1} E(Z^2_{n,3}(S)) \qquad 5.67$$

hence $\sigma^2 Z(S) = E(Z^2(S)) - (E(Z(S)))^2$ 5.68

(2) Absolute Acceleration: $\ddot{Z}(S)$

$$\ddot{Z}(S) = \sum_{n=1}^{N+1} \ddot{Z}_{n,3}(S)$$

i) Expected value of $\ddot{Z}_{n,3}(S)$: $E(\ddot{Z}(S))$

$$E(\ddot{Z}(S)) = \sum_{n=1}^{N+1} E(\ddot{Z}_{n,3}(S)) \qquad 5.69$$

ii) Variance of $\ddot{Z}(S)$: $\sigma^2 \ddot{Z}(S)$

$$E(\ddot{Z}^2(S)) = \sum_{n=1}^{N+1} E(\ddot{Z}^2_{n,3}(S)) \qquad 5.70$$

hence $\sigma \ddot{Z}^2(S) = E(\ddot{Z}^2(S)) - (E(\ddot{Z}(S)))^2$ 5.71

3) <u>Contact Force:</u> $F(S)$

$$F(S) = M\ddot{Z}(S) + (M+m)g \qquad 5.72$$

i) Expected value of F(S): E(F(S))

$$E(F(S)) = ME(\ddot{Z}(S)) + (M+m)g \qquad 5.73$$

ii) Variance of F(S): $\sigma^2 F(S)$

$$E(F^2(S)) = M^2 E(\ddot{Z}^2(S)) + 2M(M+m)g\,E(\ddot{Z}(S)) + (M+m)^2 g^2 \qquad 5.74$$

hence $\sigma^2 F(S) = E(F^2(S)) - (E(F(S)))^2$ 5.75

5.2 Results

Results obtained in this section are for vehicle described in Table 3.1, other data used are-

i) Maximum length of pulse (l_2) = 7.5m.

ii) Minimum length of pulse (l_1) = 1.5m.

iii) Minimum depth of pulse = 5mm

iv) Maximum depth of pulse = 25mm

C = 1/300.

Following observations may be made (figs. 5.3 to 5.8)

1. Expected value of absolute displacement has been plotted against S/l for different velocities in Fig.5.3. It is seen that expected values are high for low velocity. At a velocity of 30 km/hr. Maximum value is 25 mm while it is only 3.5mm for a velocity of 150km/hr.

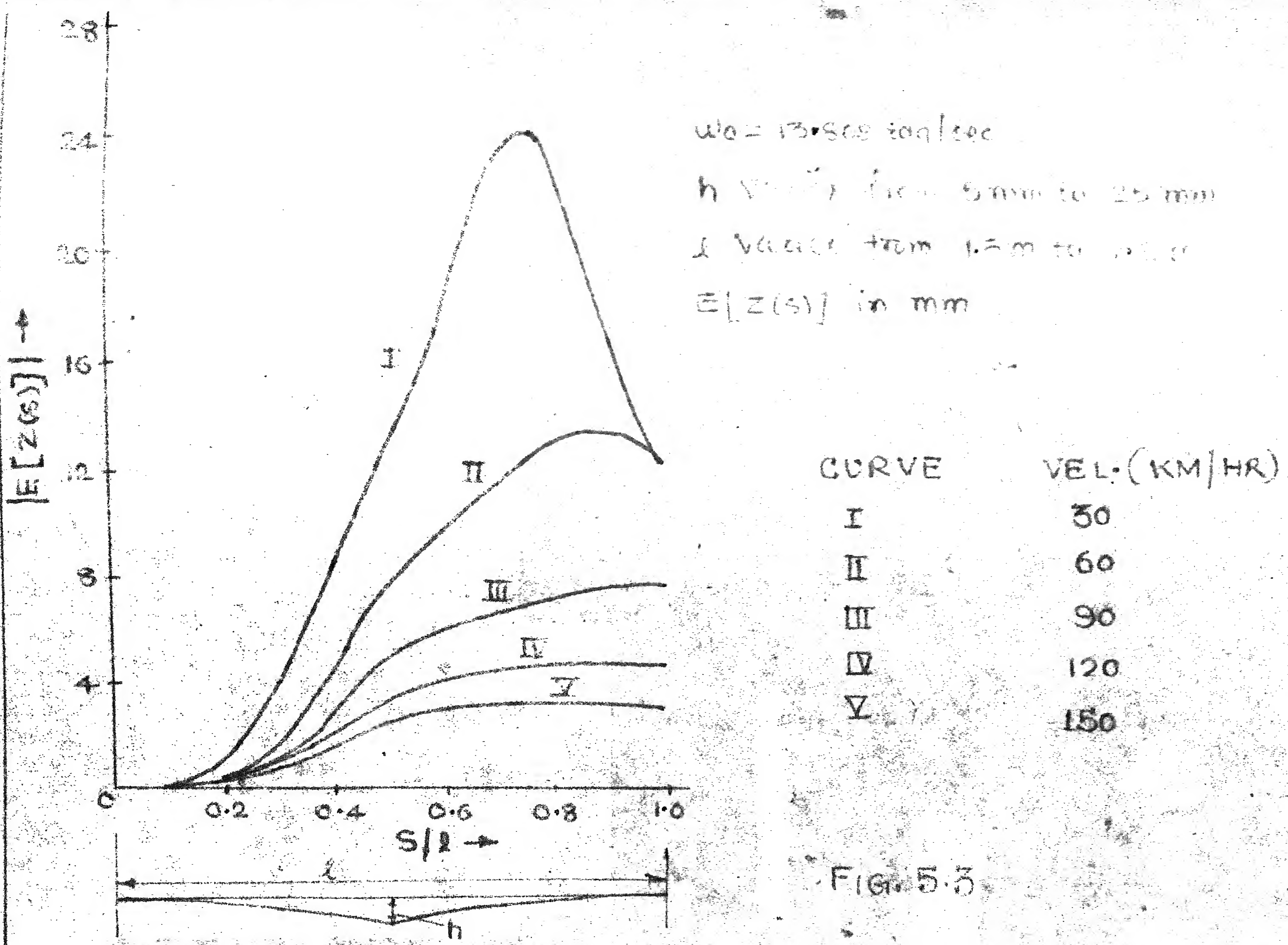
2. R.m.s. value of absolute displacement has been plotted against S/l for different velocities in Fig. 5.4. There also r.m.s. value are high for low velicity. Maximum

value of r.m.s. value at velocity of 30 km/hr is 14mm while it is 3.0 mm for a velocity of 150km/hr.

3. Ratios of expected value and r.m.s. value of absolute acceleration to the acceleration due to gravity have been plotted against S/l for different velocities in Figs. 5.5 and 5.6. It is seen that the ratios are high for high velocities. At a velocity of 30 km/hr, max value of $E(\ddot{Z})$ is about 0.06 times g and $\sigma_{\ddot{Z}}$ is 0.09 times g. For a velocity of 150 km/hr, corresponding values are 0.14 and 0.17.

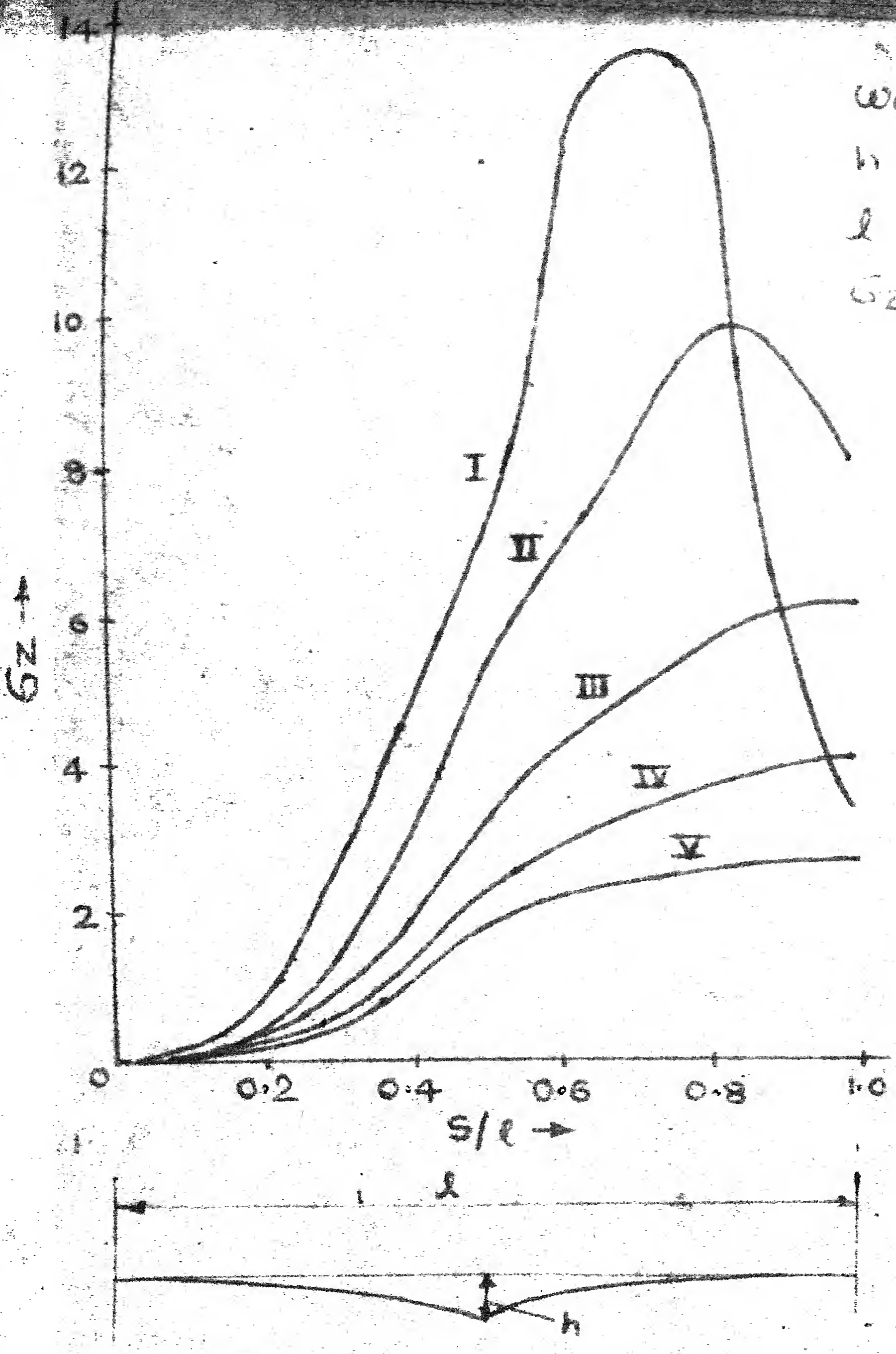
4. In Figs. 5.7 and 5.8, ratios of expected value and r.m.s. value of contact force to the total weight of the vehicle have been plotted against S/l for different velocities. It is seen that the increased velocity higher maximum value of the ratios. At a velocity of 30 km/hr, maximum values of E(F) and σ_F are 1.07 and 12 times the total weight of vehicle. For a velocity of 150 km/hr, corresponding values are 1.09 and 0.20 respectively.

Thus it is concluded that the increase velocity lowers maximum r.m.s. and expected value of absolute displacement, while opposite nature is seen in the case of acceleration and contact force i.e. increased velocity increases maximum r.m.s. and expected values.



$|E[Z(S)]|$
S/l
l
h
w_0 = 13.808 rad/sec
h varies from 5mm to 25 mm
l varies from 1.5 m to
$E[Z(S)]$ in mm
CURVE VEL.(KM/HR)
I 30
II 60
III 90
IV 120
V 150

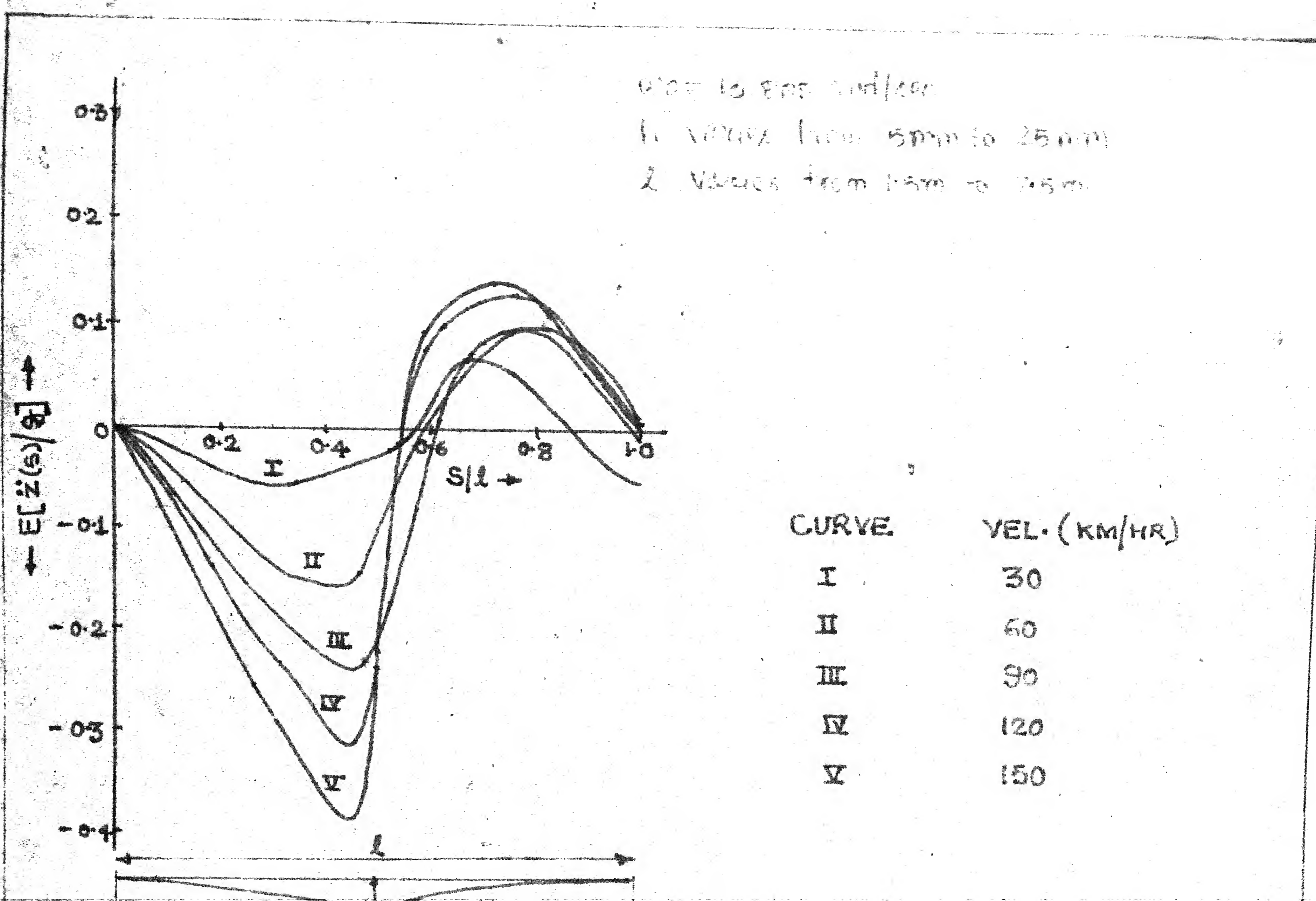
FIG. 5.3

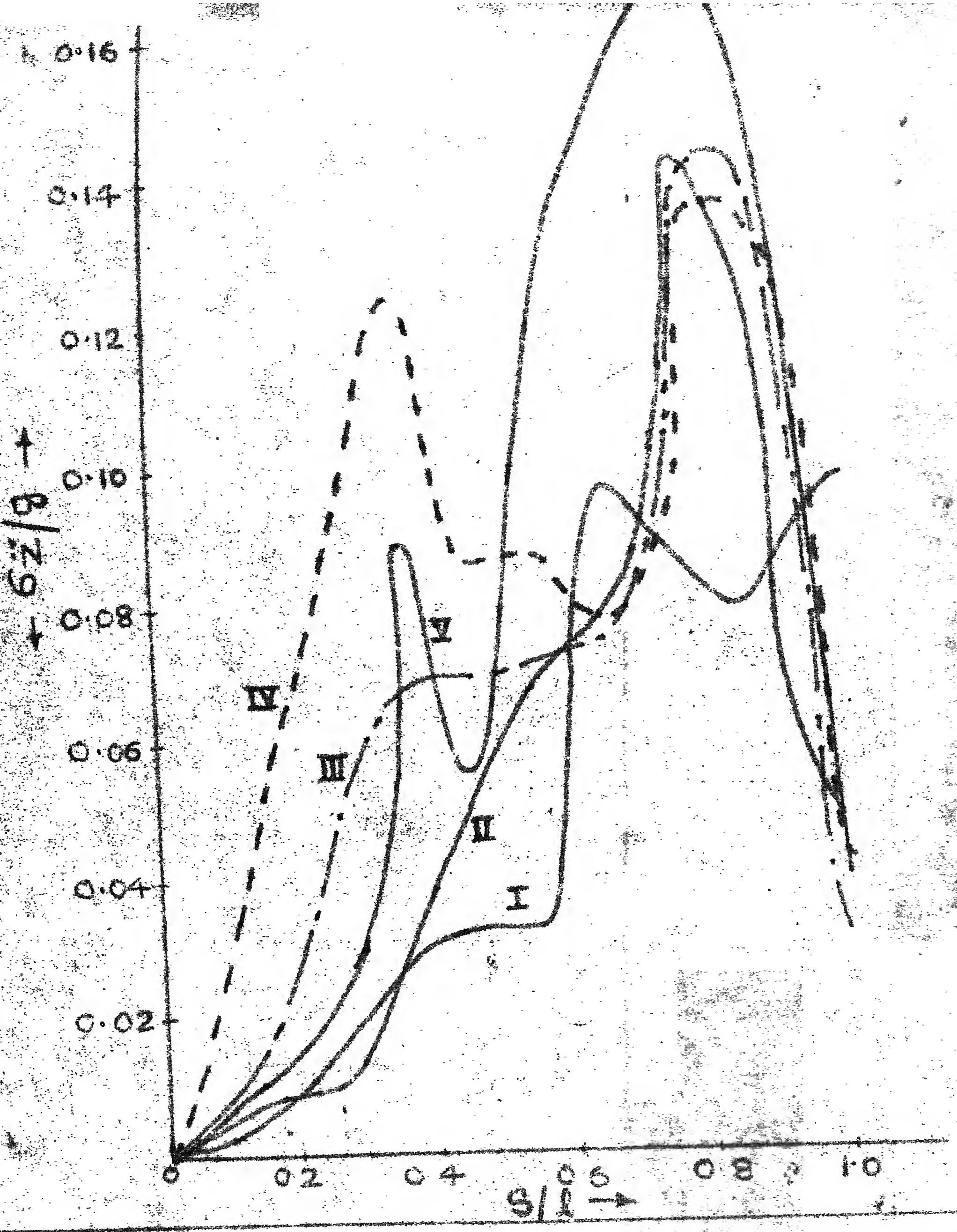


ωo = 13.808 rad/sec
h varies from 5mm to 25mm
l varies from 1.5m to 7.5m
Gz is in mm

CURVE	VEL. (KM/HR)
I	30
II	60
III	90
IV	120
V	150

FIG 5.4


h varies from 5mm to 25mm
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0.2
0.4
0.6
0.8
1.0
S/l →
← E[ż(s)/φ] →
I
II
III
IV
V
l
CURVE
VEL. (KM/HR)
I
30
II
60
III
90
IV
120
V
150



$\omega_0 = 15.808$ rad/sec

h varies from 5 mm to 25 mm

l varies from 1.5 m to 7.5 m

CURVE	VEL (KM/HR)
I	30
II	60
III	90
IV	120
V	150

FIG. 5·6

CHAPTER 6

RESPONSE OF VEHICLE TO CONTINUOUS AND TOTAL RAIL TRACK UNEVENNESS

INTRODUCTION

The continuous unevenness of the rail track can be treated as homogenous random process in space coardinates when a vehicle moves on such a track with constant velocity, the base excitation is stationary. The process is assumed to have zero mean and spectral density given by

$$\Phi_{yy}(\Omega) = \frac{c}{\Omega^2 + \Upsilon^2} \qquad |\Omega| \leq \Omega_0$$
$$= 0.$$

Where Ω is spatial frequency, c and Υ are constants. The roughness constant C is given by

$$c = \frac{\sigma^2 \Upsilon}{\pi}$$

where σ is the r.m.s. value of ground unevenness.

Second order statistics of response parameters are calculated in section 6.1. In the section 6.2, continuous unevenness is superimposed upon descrete unevenness due to random low joints. Second order statistics for total unevenness is obtained by assuming that two process are uncorrelated.

6.1 SECOND ORDER STATISTICS OF RESPONSE TO CONTINUOUS RAIL TRACK UNEVENNESS

Equation of Motion: Equation of motion for single degree of freedom model is

$$\ddot{Z} + 2\zeta\omega_0\dot{Z} + \omega_0^2 Z = 2\zeta\omega_0\dot{Y} + \omega_0^2 Y \qquad 6.1$$

for constant velocity motion S = Vt, above equation becomes,

$$V^2 Z'' + 2\zeta V\omega_0 Z' + \omega_0^2 Z = 2\zeta\omega_0 VY' + \omega_0^2 Y \qquad 6.2$$

Solution of equation of motion:

The solution of equation 6.2 can be expressed as

$$Z(S) = \sum_{i=1}^{2} \beta_i e^{\frac{\alpha_i s}{V}} + \int_{-\infty}^{\infty} dF(\Omega) e^{J\Omega S} H(\Omega) \qquad 6.3$$

where

$$\alpha_i = (-\zeta \pm \sqrt{\zeta^2 - 1})\,\omega_o \quad i = 1,2$$

$$H(\Omega) = \frac{(2\zeta\omega_0 JV\Omega + \omega_0^2)}{(\omega_0^2 - V^2\Omega^2 + 2J\zeta\Omega V\omega_0)} \qquad 6.4$$

β_i are constants and to be determined from initial conditions.

$$Y(S) = \int_{-\infty}^{\infty} e^{j\Omega s}\, dF(\Omega) \qquad 6.5$$

and $F(\Omega)$ is related to the P.S.d function of $Y(S)$ by the following equations

$$E(|dF(\Omega)|^2) = \Phi_{yy}(\Omega)\, d\Omega \qquad 6.6$$

also $E(dF(\Omega)) = 0$ 6.7

as $E(Y(S)) = 0$ 6.8

for zero initial conditions, we can write as

$$Z(S) = \int_{-\infty}^{\infty} dF(\Omega) e^{j\Omega S} H(\Omega) \qquad 6.9$$

$$\text{and } \ddot{Z}(S) = -V^2 \int_{-\infty}^{\infty} \Omega^2 e^{j\Omega S}\, dF(\Omega) H(\Omega) \qquad 6.10$$

1) Expected value of $Z(S)$ and $\ddot{Z}(S)$: $E(Z(S))$, $E(\ddot{Z}(S))$

$$E(Z)(S)) = \int_{-\infty}^{\infty} H(\Omega)e^{J\Omega S} \; E(dF(\Omega)) = 0$$

from eq ation 6.7, and

$$E(\ddot{Z}(S)) = V^2 \frac{d^2 E}{dS^2} (Z(S)) = 0 \text{ from above.}$$

(2) Covariance of $Z(S)$ and $\ddot{Z}(S)$:

The response and its defivaties can be closed as an oscillary process. Hence the evolutionary spectral density, which is the common spectral density in present case can be determined by the use offollowing expression given by Priestley.

$$\Phi_{zz}(\Omega) = \Phi_{yy}(\Omega)\, H(\Omega)\, H^*(\Omega) \qquad 6.11$$

Where (*) denotes conjugate operator.

As the eigen values and eigen vectors occur in complex conjugate, above expression can be written in a simpler form.

$$\Phi_{ZZ}(\Omega) = \Phi_{yy}(\Omega)H(\Omega)H(-\Omega) \qquad 6.12$$

covariance function for the response is

$$K_{zz}(S1,\ S2) = \int_{-\infty}^{\infty} \Phi_{yy}(\Omega)H(\Omega)H^*(\Omega)e^{J\Omega(S1-S2)}d\Omega$$

with the same argument, the conjugate operator may be dropped with change in sign of . Hence

$$K_{zz}(S1,\ S2) = \int_{-\infty}^{\infty} \Phi_{yy}(\Omega)H(\Omega)H(-\Omega)\; e^{j\Omega(S1-S2)}d\Omega \qquad 6.13$$

Substituting values of $H(\Omega)$, $H(-\Omega)$ and $\Phi_{yy}(\Omega)$ we get,

$$K_{zz}(s1,S2)=\int_{-\infty}^{\infty}\left(\frac{c}{\Omega^2+\gamma^2}\,\frac{(2\zeta\omega_0 vJ\Omega+\omega_0^2)(-2\zeta\omega_0 vJ\Omega+\omega_0^2))}{(\omega_0^2-V^2\Omega^2+2\zeta\omega_0 vJ\Omega)(\omega_0^2-V^2\Omega^2-2\zeta\omega_0 Vj\Omega)}\right.$$

$$\cdot\, e^{j\Omega(S1-S2)}d\Omega \qquad 6.14$$

The improper integral above can be evaluated by contour integration with residue theorem. There are two poles in the upper part of complex domain at $\Omega = J\gamma$ and and $= \frac{\alpha_i}{JV}$ $i = 1,2$

$$\alpha_1 = (-\zeta - \sqrt{\zeta^2-1})\,\omega_0$$

$$\alpha_2 = (-\zeta + \sqrt{\zeta^2-1})\,\omega_0$$

both the poles are of first order. Hence

$$K_{zz}(S1,S2) = C2\pi J\left(\frac{(\omega_0^4-4\zeta^2V^2\omega_0^2\gamma^2\; e^{-\gamma(S1-S2)})}{(2J\gamma(\omega_0^2+V^2\gamma^2-2\zeta\omega_0V\gamma)(\omega_0^2+V^2\gamma^2+2\zeta\omega_0V\gamma))}\right.$$

$$+\frac{e^{\alpha_1\left|\frac{S1-S2}{V}\right|}(\omega_0^4-4\zeta^2\omega_0^2\alpha_1^2)}{(\gamma^2-\frac{\alpha_1^2}{V^2})(\omega_0^2+\alpha_1^2-2\zeta\omega_0\alpha_1)(2J\sqrt{\zeta^2-1})\,\omega_0V}$$

$$\left.+\frac{e^{\alpha_2\left|\frac{S1-S2}{V}\right|}(\omega_0^4-4\zeta^2\omega_0^2\alpha_2^2)}{(\gamma^2-\frac{\alpha_2^2}{V^2})(\omega_0^2+\alpha_2^2-2\zeta\omega_0\alpha_2)(-2J\sqrt{\zeta^2-1})\,\omega_0V}\right)$$

$$=\pi C\left(\frac{(\omega_0^4-4\zeta^2V^2\omega_0^2\gamma^2)\;e^{-\gamma|S1-S2|}}{(\gamma((\omega_0^2+V^2\gamma^2)^2-4\zeta^2\omega_0^2V^2\gamma^2))}\right.$$

$$+\frac{e^{\frac{\alpha_1|S1-S2|}{V}}\left(\omega_0^4-4\zeta^2\omega_0^2\alpha_1^2\right)}{\left(\gamma^2-\frac{\alpha_1^2}{V^2}\right)\left(\omega_0^2+\alpha_1^2-2\zeta\omega_0\alpha_1\right)\left(\zeta^2-1\right)^{\frac{1}{2}}\omega_0 V}$$

$$-\frac{e^{\frac{\alpha_2|S1-S2|}{V}}\left(\omega_0^4-4\zeta^2\omega_0^2\alpha_2^2\right)}{\left(\gamma^2-\frac{\alpha_2^2}{V^2}\right)\left(\omega_0^2+\alpha_2^2-2\zeta\omega_0\alpha_2\right)\left(\zeta^2-1\right)^{\frac{1}{2}}\omega_0 V}\Bigg)\Bigg)$$

Above equation indicates that $k_{zz}(S1,S2)$ is a function of $S1-S2$. Hence displacement and its time derivatives are homogenous process. In that case,

$$R_{zz}(S1-S2) = K_{zz}(S1,S2)$$

also if $S1-S2 = S$ then

$$R_{zz}(S) = \pi C \Bigg\{ \frac{\left(\omega_0^4-4\zeta^2V^2\omega_0^2\gamma^2\right)e^{-\gamma|S|}}{\left(\gamma\left\{\omega_0^2+V^2\gamma^2\right\}^2-4\zeta^2\omega_0^2V^2\gamma^2\right)}$$

$$+\frac{e^{\frac{\alpha_1|S|}{V}}\left(\omega_0^4-4\zeta^2\omega_0^2\alpha_1^2\right)}{\left(\gamma^2-\frac{\alpha_1^2}{V^2}\right)\left(\omega_0^2+\alpha_1^2-2\zeta\omega_0\alpha_1\right)\left(\zeta^2-1\right)^{\frac{1}{2}}\omega_0 V}$$

$$-\frac{e^{\frac{\alpha_2|S|}{V}}\left(\omega_0^4-4\zeta^2\omega_0^2\alpha_2^2\right)}{\left(\gamma^2-\frac{\alpha_2^2}{V^2}\right)\left(\omega_0^2+\alpha_2^2-2\zeta\omega_0\alpha_2\right)\left(\zeta^2-1\right)^{\frac{1}{2}}\omega_0 V}\Bigg\}$$

Therefore

$$E(Z^2(S)) = R_{zz}(o) = \pi C \Bigg\{ \frac{\left(\omega_0^4-4\zeta^2V^2\omega_0^2\gamma^2\right)}{\gamma\left(\left(\omega_0^2+V^2\gamma^2\right)^2-4\zeta^2V^2\omega_0^2\gamma^2\right)}$$

$$+\frac{\left(\omega_0^4-4\zeta^2\omega_0^2\alpha_1^2\right)}{\left(\gamma^2-\frac{\alpha_1^2}{V^2}\right)\left(\omega_0^2+\alpha_1^2-2\zeta\omega_0\alpha_1\right)\left(\zeta^2-1\right)^{\frac{1}{2}}\omega_0 V}$$

$$- \frac{(\omega_0^4 - 4\zeta^2\omega_0^2\alpha_2^2)}{(\gamma^2 - \frac{\alpha_2^2}{v^2})(\omega_0^2 + \alpha_2^2 - 2\zeta\omega_0\alpha_2)(\zeta^2-1)^{\frac{1}{2}}\omega_0 v} \Big\} \quad 6.16$$

Now variance $\sigma^2(S)$ is

$$\sigma_Z^2(S) = E(Z^2(S)) - ((E(Z(S)))^2$$
$$= E(Z^2(S)); \; 6.17; \text{ as } E(Z(S)) \text{ is zero.}$$

Since $K_{zz}(S1,S2)$ is a function of $(S1-S2)$, then for evaluation of the covariance function of the second derivatives, the differentiation has to be taken with respect to $(S1-S2)$. Hence

$$K_{\ddot{z}\ddot{z}}(S1,S2) = v^4 \frac{\partial^4}{\partial(S1-S2)^4} K_{zz}(S1-S2)$$

$= R_{\ddot{Z}\ddot{Z}}(S)$ where $S = S1-S2$

$$= v^4 \int_{-\Omega_0}^{\Omega_0} \frac{C\Omega^4}{(\Omega^2+\gamma^2)} \left\{ \frac{(2\zeta\omega_0 V\Omega J + \omega_0^2)(-2\zeta\omega_0 V\Omega J + \omega_0^2)e^{J\Omega(S1-S2)}}{(\omega_0^2 - V^2\Omega^2 + 2\zeta\omega_0 VJ\Omega)(\omega_0^2 - V^2\Omega^2 - 2\zeta VJ\omega_0\Omega)} \right\} d\Omega. \quad 6.18$$

Therefore, $E(\ddot{Z}^2(S)) = R_{\ddot{Z}\ddot{Z}}(0)$ 6.19

Hence variance $\sigma_{\ddot{Z}}^2(S)$:

$$\sigma_{\ddot{Z}}^2(S) = E(\ddot{Z}^2(S)) - \{E(\ddot{Z}(S))\}^2$$

$= E\{\ddot{Z}^2(S)\}$ as $E(\ddot{Z}(S))$ is zero. 6.20

3) Covariance of $\ddot{Y}(S)$ and $\ddot{Z}(S)\ddot{Y}(S)$:

We have

$$K_{YY}(S1,S2) = \int_{-\infty}^{\infty} \Phi_{YY}(\Omega)e^{J\Omega(S1-S2)}\, d\Omega$$

$$= C \int_{-\infty}^{\infty} \frac{1}{\Omega^2 + \Upsilon^2} \; e^{J\Omega(S1-S2)} d\Omega \qquad 6.21.$$

also $K_{\ddot{Y}\ddot{Y}}(S1,S2) = V^4 \dfrac{\partial^4}{\partial S_1^2 \, \partial S_2^2} K_{YY}(S1,S2)$

but sin

$K_{\ddot{Y}\ddot{Y}}$(S

$= V^4$

But $K_{\dot{Y}\dot{Y}}$

Therefo

and E($\dot{}$

Therefo

$\sigma_{\ddot{Y}}^{\,2}$(S)

= E ($\dot{Y}$

Again w

$Z(S_1)$ =

$Y(S_2)$ =

hence

$= \int_{-\infty}^{\infty}$

$= \int_{-\infty}^{\infty}$ (

Since K_{ZY} is also a function of (S1-S2) therefore

$$K_{\ddot{Z}\ddot{Y}}(S1,S2) = R_{\ddot{Z}\ddot{Y}}(S1-S2) = V^4 \frac{\partial^4}{\partial(S1-S2)^4} K_{ZY}(S1-S2)$$

$$= V^4 \int_{-\Omega_0}^{\Omega_0} \frac{C\,\Omega^4}{\Omega^2+\gamma^2} \frac{(2\zeta\omega_0 V\Omega J + \omega_0^2)e^{J\Omega(S1-S2)}}{(\omega_0^2 - V^2\Omega^2 + 2\zeta\omega_0 JV\Omega)} d\Omega$$

hence $$R_{\ddot{Z}\ddot{Y}}(S) = V^4 \int_{-\Omega_0}^{\Omega_0} \frac{C\,\Omega^4}{\Omega^2+\gamma^2} \frac{(2\zeta\omega_0 VJ\Omega + \omega_0^2)e^{J\Omega S}}{(\omega_0^2 - V^2\Omega^2 + 2\zeta\omega_0 JV\Omega)} d\Omega \qquad 6.27$$

(d) Expected value and variance of contact force F(S):

$$F(S) = M\ddot{Z}(S) + m\ddot{Y}(S) + (M+m)g \qquad 6.28$$

hence

i) Expected value of F(S) = E(F(S)):

$$E(F(S)) = ME(\ddot{Z}(S)) + mE(\ddot{Y}(S)) + (M+m)g = (M+m)g \qquad 6.29$$

as $E(\ddot{Z}(S))$ and $E(\ddot{Y}(S))$ are zero.

ii) Variance of F(S) : $\sigma_F^2(S)$

$$E(F^2(S)) = M^2E(\ddot{Z}^2(S)) + m^2E(\ddot{Y}^2(S)) + (M+m)^2g^2$$

$$+2MmE(\ddot{Z}(S)\ddot{Y}(S)) + 2(M+m)Mg\,E(\ddot{Z}(S)) + 2(M+m)mg\,E(\ddot{Y}(S))$$

$$= M^2E(\ddot{Z}^2(S)) + m^2E(\ddot{Y}^2(S)) + (M+m)^2g^2 + 2MmE(\ddot{Z}(S)\ddot{Y}(S)) \qquad 6.30$$

by substituting values of $E(\ddot{Z}^2(S))$, $E(\ddot{Y}^2(S))$ and $E(\ddot{Z}(S)\ddot{Y}(S))$ from above, $E(F^2(S))$ can be calculated,

hence, $\sigma_F^2(S) = E(F^2(S)) - \{E(F(S))\}^2$

$$= M^2E(\ddot{Z}^2(S)) + m^2E(\ddot{Y}^2(S)) + 2MmE(\ddot{Z}(S)\ddot{Y}(S)) \qquad 6.30$$

6.2 SECOND ORDER STATISTICS OF RESPONSE PARAMETERS TO TOTAL UNEVENNESS:

In the last section, we have obtained second order statistics due to continuous unevenness only. In this section, continuous unevenness is superimposed upon descrete unevenness due to low joints. Second order statistics of response process due to total unevenness is obtained by assuming the two processes are uncorrelated.

Let $Y_1(S)$ be the process due to continuous unevenness of rail track, treated as homogenous process with zero mean.

$Z_1(S)$ the response process due to $Y_1(S)$ and can also be treated as homogenous process with zero mean.

$Y_2(S)$ be the process due to discrete unevenness of random rail joints.

$Z_2(S)$ be the response process due to $Y_2(S)$

By superpositioning on principle

$Z(S) = Z_1(S) + Z_2(S)$

and $Y(S) = Y_1(S) + Y_2(S)$

1) Second order Statistics of $Z(S)$:

i) Expected value of $Z(S)$: $E(Z(S))$

$$E(Z(S)) = E(Z_1(S)) + E(Z_2(S)) = E(Z_2(S)) \qquad 6.31$$

as $E(Z_1(S)) = 0$.

ii) Variance of $Z(S)$: $\sigma^2 Z(S)$

$$E(Z^2(S)) = (E(Z_1^2(S)) + (E(Z_2^2(S)) + E(Z_1(S)Z_2(S)) \qquad 6.32$$

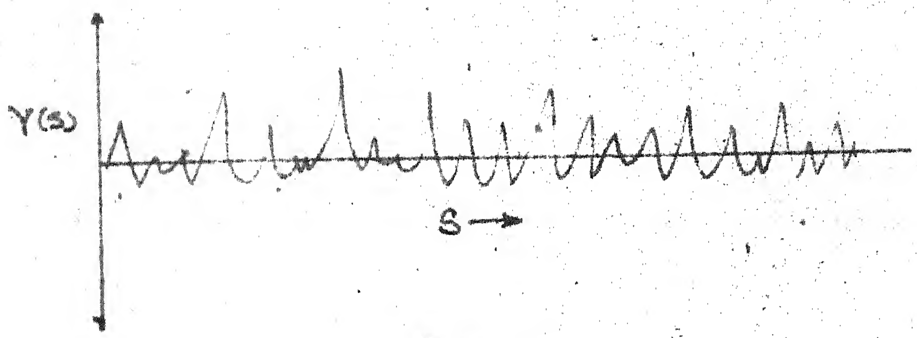


FIG. 6·1

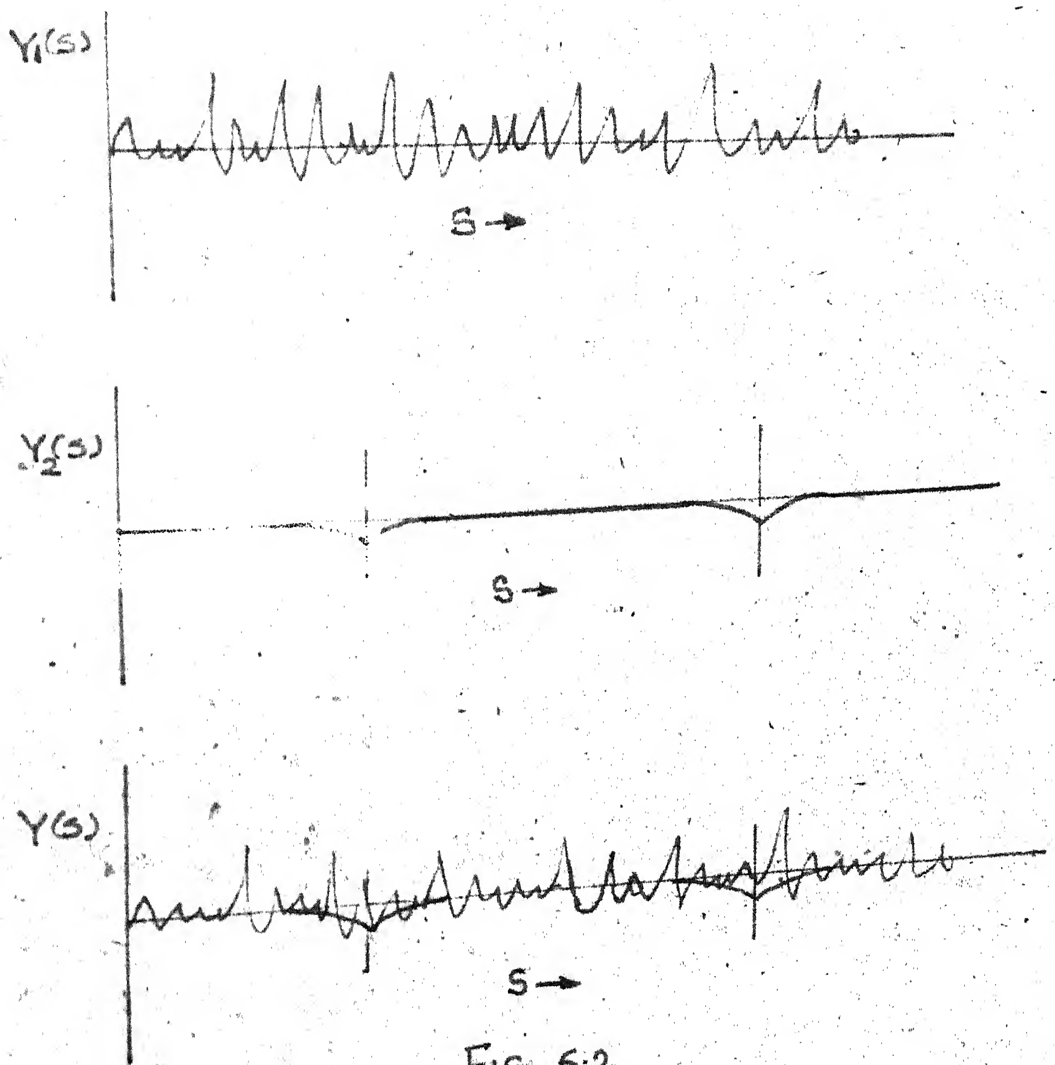


FIG. 6·2

$$= E(Z_1^2(S)) + E(Z_2^2(S)) \qquad 6.33$$

$E\ (Z_1(S)\ Z_2(S)) = 0$ as the two processes have been assumed to be uncorrelated

$$\sigma Z^2(S) = E(\ Z^2(S)) - [E(Z(S))]^2$$

$$= E(Z_1^2(S)) + E(Z_2^2(S)) - [E(Z_2(S))]^2$$

$$= E(Z_1^2(S)) + \sigma^2 Z_2(S) \qquad 6.34$$

$$\text{where}\quad \sigma Z_2^2(S) = E(Z_2^2(S)) - [E(Z_2(S))]^2 \qquad 6.35$$

2) <u>Second Order Statistics of</u> $\ddot{Z}(S)$:

$$\ddot{Z}(S) = \ddot{Z}_1(S) + \ddot{Z}_2(S)$$

i) Expected value of $\ddot{Z}(S)$: $E(\ddot{Z}(S))$

$$E(\ddot{Z}(S)) = E(\ddot{Z}1(S)) + E(\ddot{Z}_2(S))$$

$$= E(\ddot{Z}_2(S)) \text{ as } E(\ddot{Z}_1(S)) = 0. \qquad 6.36$$

ii) Va iance of $\ddot{Z}(S)$: $\sigma\ddot{Z}^2(S)$

$$E(\ddot{Z}^2(S)) = E(\ddot{Z}_1^2(S)) + E(\ddot{Z}2(S)) + 2E(\ddot{Z}_1(S)\ \ddot{Z}_2(S))$$

$$= E(\ddot{Z}_1^2(S)) + E(\ddot{Z}_2^2(S)) \qquad 6.37$$

$$\text{hence}\ \sigma\ddot{Z}^2(S) = E(\ddot{Z}^2(S)) - [E(\ddot{Z}(S))]^2$$

$$= E(\ddot{Z}_1^2(S)) + E(\ddot{Z}_2^2(S)) - [E(\ddot{Z}_2(S))]^2$$

$$= E(\ddot{Z}_1^2(S)) + \sigma\ddot{Z}_2^2(S) \qquad 6.38$$

$$\text{where}\quad \sigma^2\ddot{Z}_2(S) = E(\ddot{Z}_2^2(S)) - [E(\ddot{Z}_2(S))]^2$$

(3) Second Order Statistics of Contact Force F(S)

$F(S) = M\ddot{Z}(S) + m\ddot{Y}(S) + (M+m)g$

i) Expected value of F(S): E(F(S))

$$E(F(S)) = ME(\ddot{Z}(S)) + mE(\ddot{Y}(S)) + (M+m)g \qquad 6.39$$

where $E(\ddot{Z}(S))$ as calculated above, and

$E(\ddot{Y}(S)) = E(\ddot{Y}_1(S)) + E(\ddot{Y}_2(S)) = E(\ddot{Y}_2(S))$ as

$$E(\ddot{Y}_1(S)) = 0 \qquad 6.40$$

(ii) Variance of F(S): $\sigma^2 F(S)$

$E(F^2(S)) = M^2E(\ddot{Z}^2(S)) + m^2E(\ddot{Y}^2(S)) + (M+m)^2g^2$

$+2M(M+m)g\ E(\ddot{Z}(S)) + 2mME(\ddot{Z}(S)\ddot{Y}(S)) +$

$$2m(M+m)g\ E(\ddot{Y}(S)) \qquad 6.41$$

where

$$E(\ddot{Z}^2(S)) = E(\ddot{Z}_1^2(S)) + E(\ddot{Z}_2^2(S)) \qquad 6.42$$

$E(\ddot{Y}^2(S)) = E(\ddot{Y}_1^2(S)) + E(\ddot{Y}_2^2(S)) + 2E(\ddot{Y}_1(S)\ddot{Y}_2(S))$

$$= E(\ddot{Y}_1^2(S)) + E(\ddot{Y}_2^2(S)) \qquad 6.43$$

as $\ddot{Y}_1(S)$ and $\ddot{Y}_2(S)$ are uncorrelated process and therefore

$E(\ddot{Y}1(S)\ddot{Y}_2(S)) = 0$

$$E(\ddot{Z}(S)) = E(\ddot{Z}_2(S)) \qquad 6.44$$

$$E(\ddot{Y}(S)) = E(\ddot{Y}_2(S)) \qquad 6.45$$

$$E(\ddot{Z}(S)\ddot{Y}(S)) = E(\ddot{Z}_1(S)\ddot{Y}_1(S) + \ddot{Z}_1(S)\ddot{Y}_2(S) + \ddot{Z}_2(S)$$

$$\ddot{Y}_1(S) + \ddot{Z}_2(S)\ddot{Y}_2(S))$$

$$= E(\ddot{Z}_1(S)\ddot{Y}_1(S)) + E(\ddot{Z}_2(S)\ddot{Y}_2(S)) \qquad 6.46$$

$$\text{hence } \sigma_F^2(S) = E(F^2(S)) - \{E(F(S))\}^2 \qquad 6.47$$

6.3. RESULTS

Results obtained in this section are for a vehicle, the data for which is given in Table 3.1. In addition to that other data used are

Damping coefficient $\zeta \doteq 0.2$

Roughness constant $C = 0.00005$

$\gamma = 0.1.$

Following observations may be made (Figs. 6.3 to 6.8)

1. In Fig. 6.3, ratio of the r.m.s. value of absolute displacement to the r.m.s. value of ground unevenness has been plotted against velocity. It is seen that the ratio increases linearly with velocity being about 1.2 times the r.m.s. value of Y at a velocity of 150km/hr.

2. In Fig. 6.4, ratio of the r.m.s. value of absolute acceleration to the acceleration due to gravity g, has been plotted against velocity. At low velocity, ratio increases sharply, but for higher velocity (> 60km/hr). Variation is approximately linear. At a velocity of 150km/hr, $\sigma_{\ddot{Z}}$ is 0.27 times acceleration due to gravity g.

3) In Fig. 6.5 ratio of the r.m.s. value of contact force to the total weight of vehicle has been plotted against velocity ratio increases very sharply at low velocity. At a velocity of 150km/hr, σ_F is 0.24 times the total weight of vehicle.

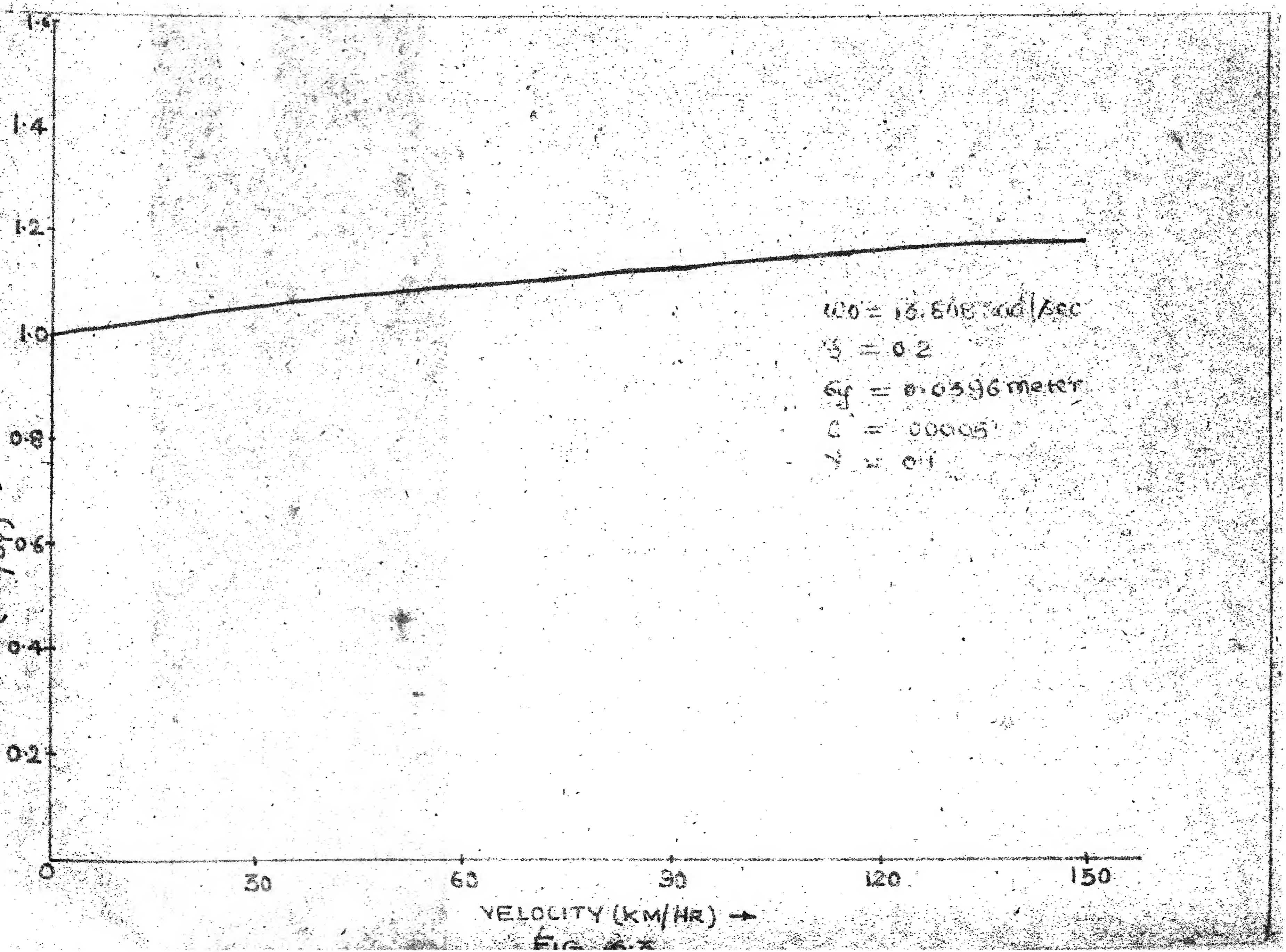
4) Assuming the unevenness to be normally distributed the 3σ values which have the excedence probability of 0.0027 are $3\sigma_Z$, $3\sigma_{\ddot{Z}}$ and $3\sigma_F$. 3σ values have been plotted in Figs. 6.6, 6.7 and 6.8 against velocity.

5) When discrete unevenness due to low joints is superimposed upon continuous unevenness (section 6.2), the second order statistics of the resulting process is obtained by assuming the two processes are uncorrelated. Since the expected values of displacement and acceleration process due to continuous unevenness are zero, the expected values of displacement and acceleration process due to total unevenness are same as that due to discrete unevenness. R.m.s. value of the total response process is obtained by using following relation.

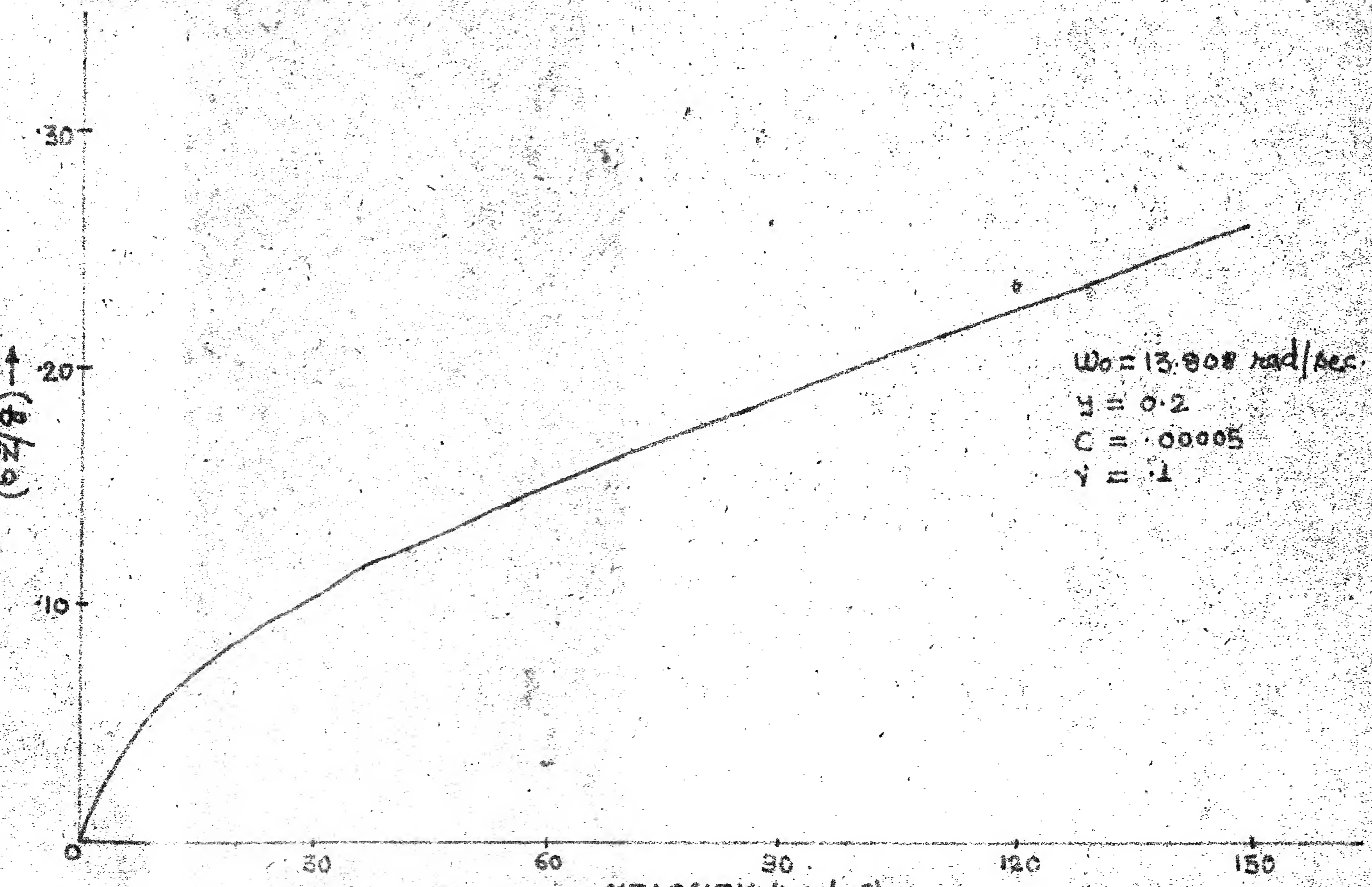
$$\sigma = (\sigma_1^2 + \sigma_2^2)^{1/2}$$

where σ_1 and σ_2 are the r.m.s. value of response process due to descrete and continuous unevenness. Expected value of contact force due to continuous unevenness is the weight of vehicle. R.m.s. value of total response process has been plotted in Figs. 6.9, 6.10 and 6.11, against S/l for velocities of 30 and 60 km/hr. It is seen that increase velocity gives higher values.

Thus it is concluded that in the continuous unevenness, r.m.s. values of acceleration and contact force increases with velocity. Results very much depend upon chosen value of roughness constant C. Expected value of total response process is same as that of response process due to discrete unevenness. R.m.s. value of total response process increases with velocity.



1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
30
60
90
120
150
VELOCITY (KM/HR) →
ω0 = 13.808 rad/sec
ξ = 0.2
σf = 0.0396 meter
C = .00005
γ = 0.1



0
·10
·20
·30
30
60
90
120
150
VELOCITY (KM/HR)
Wo = 13.808 rad/sec
y = 0.2
C = ·00005
Y = ·1

FIG 6.4

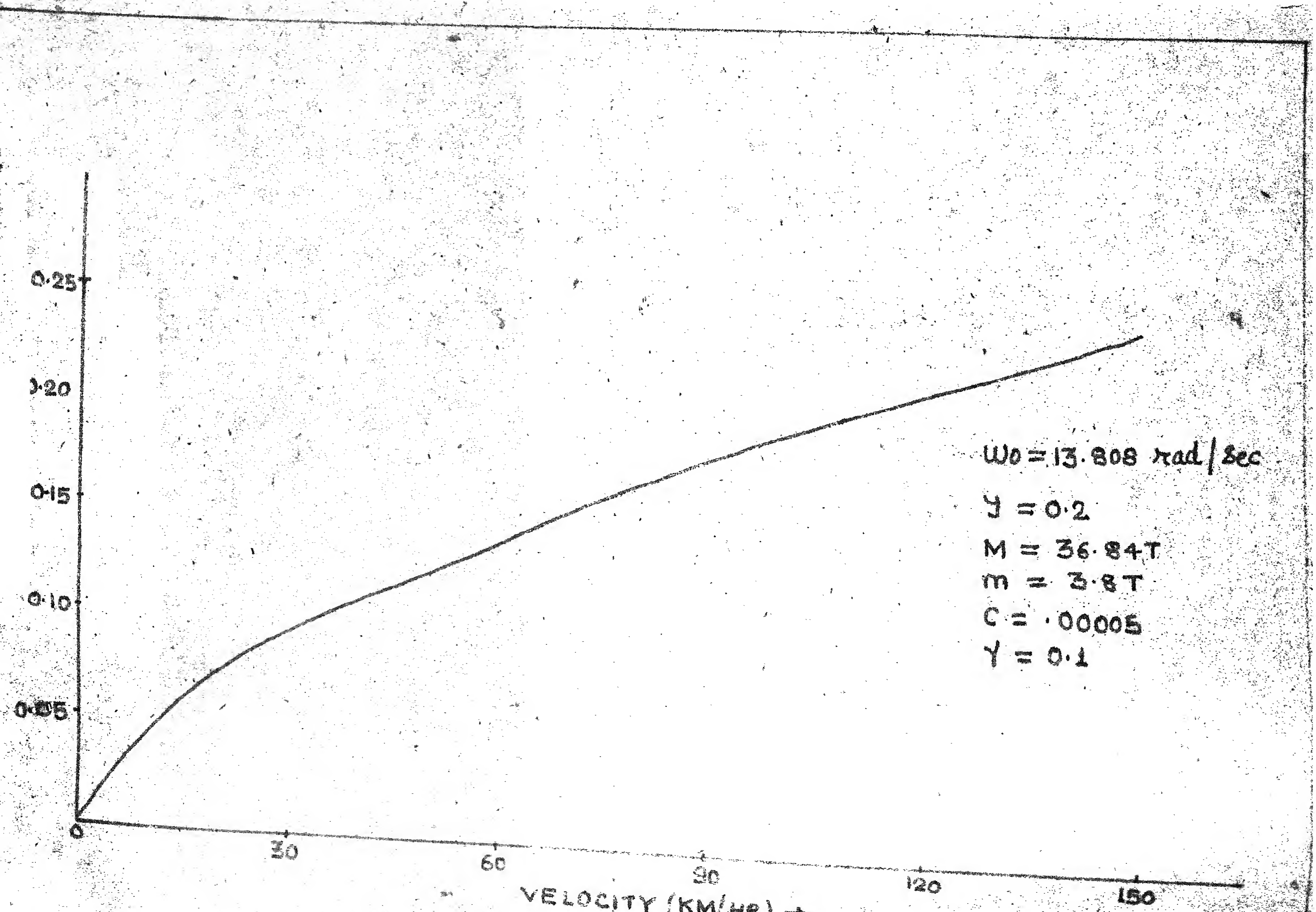

0.25
0.20
0.15
0.10
0.05
0
30
60
90
120
150
VELOCITY (KM/HR) →
Wo = 13.808 rad/sec
y = 0.2
M = 36.84T
m = 3.8T
C = ·00005
Y = 0.1

FIG. 6.8

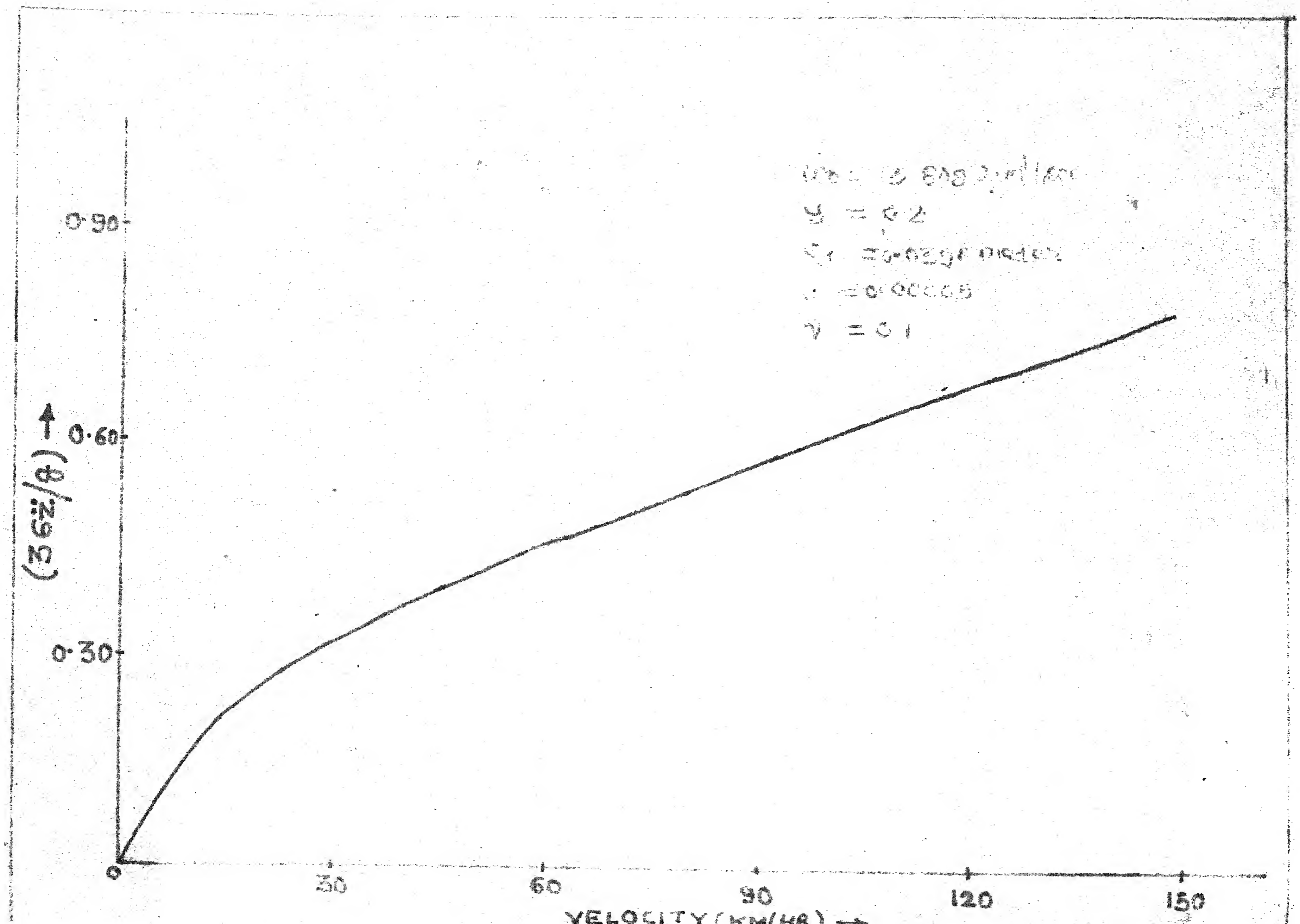
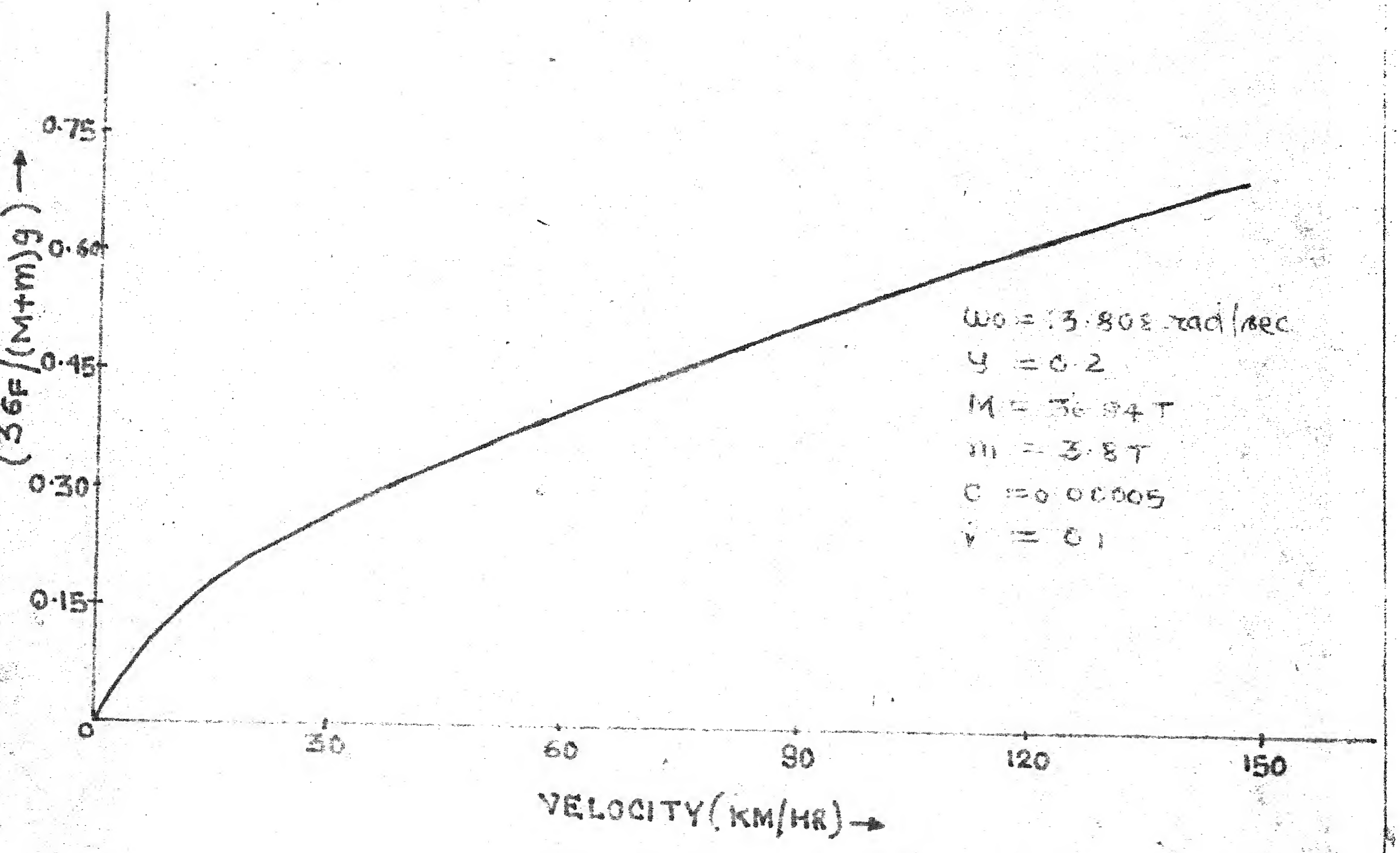
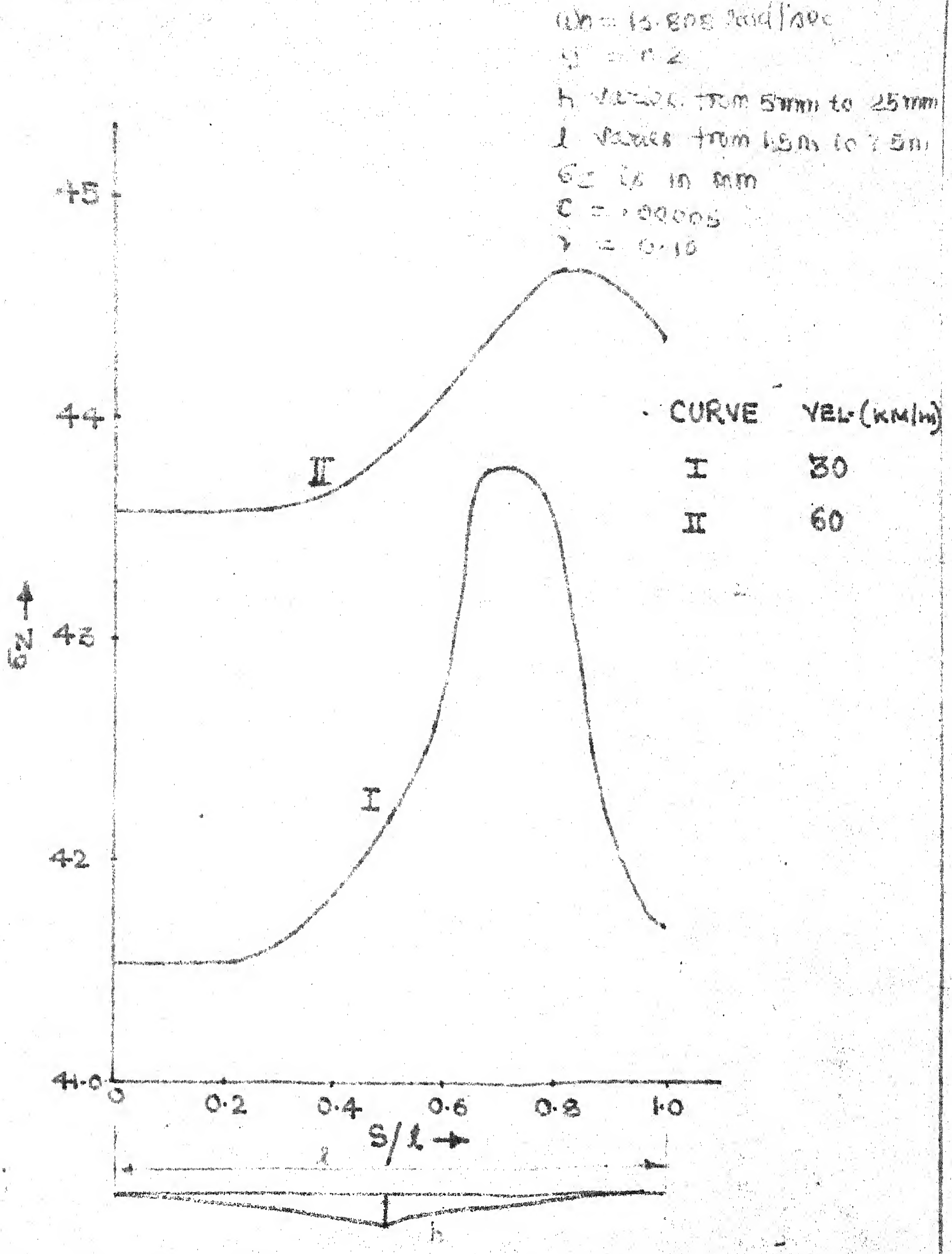


FIG. 6.7



(36F/(M+m)g) →
0.75
0.60
0.45
0.30
0.15
0
30
60
90
120
150
VELOCITY (KM/HR) →
ωo = 13.808 rad/sec
y = 0.2
M = 36.84 T
m = 3.8 T
C = 0.00005
v = 0.1

FIG. 6.8



ωn = 15.808 rad/sec
h varies from 5mm to 25mm
l varies from 1.5m to 7.5m
σz is in mm
C = .00005
γ = 0.10
CURVE VEL (KM/H)
I 30
II 60
II
I
σz
45
44
43
42
41.0
0
0.2
0.4
0.6
0.8
1.0
S/l
l
h

FIG. 6.9

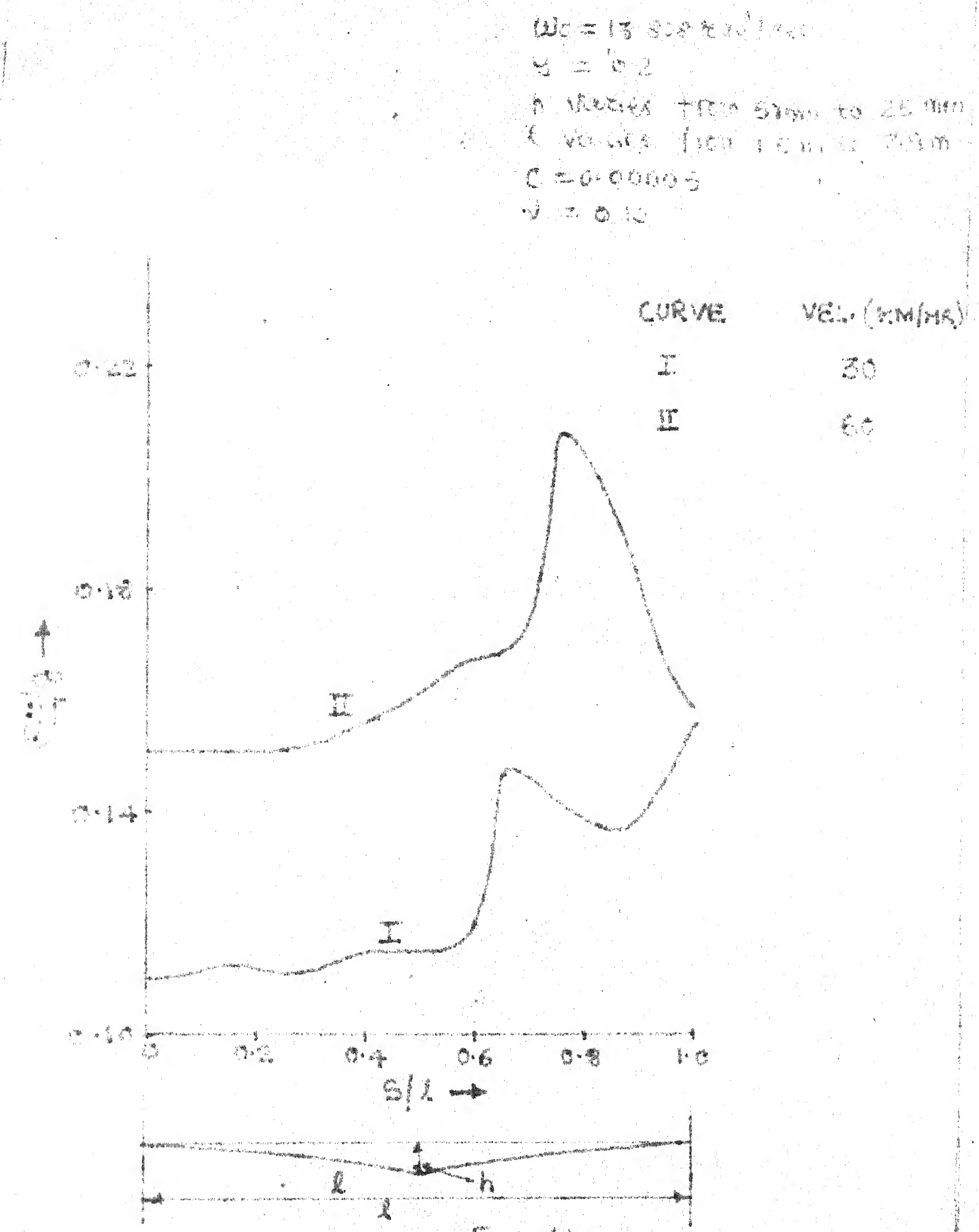

C = 0.00005
CURVE
VEL. (KM/HR)
I
30
II
60
0.22
0.18
0.14
0.10
0
0.2
0.4
0.6
0.8
1.0
S/l →
II
I
l
h

FIG. 6.10

[illegible] = 12 [illegible]

[illegible]

h varies from 5mm to 25mm

l varies from 1.5m to 7.5m

[illegible]

[illegible]

M = [illegible]

m = [illegible]

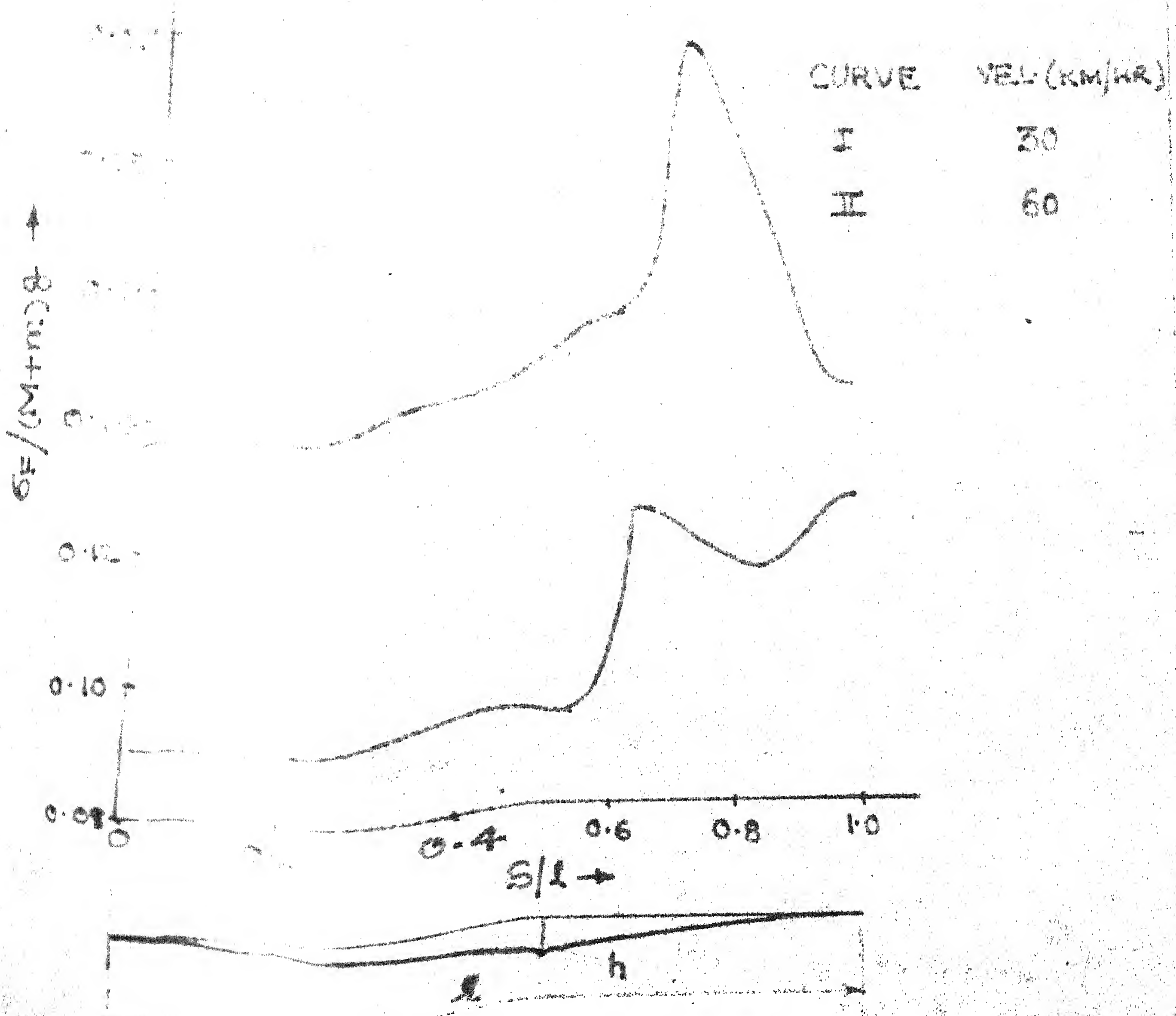


FIG. 6·11

CHAPTER 7

SUMMARY AND CONCLUSIONS

7.1 SUMMARY

A vehicle moving in contact with the ground follows the ground profile. The ground profile, therefore acts as a base excitation for the vehicle through the points of contact. Present investigation deals with the response of railway wagons to the vertical unevenness of the rail track. Two types of unevenness of the track have been considered. First continuous unevenness due to the irregularities of the rail profile and second, unevenness due to hogging of the rails at the joints. The vibration effects of each type of the unevenness have been analysed separately and superimposed.

Idealised models have been constructed for vehicle and track unevenness. It has been assumed that vehicle moves with constant velocity. The governing equations of motion have been decoupled to give set of first order differential equations.

Response statistics for periodically occurring low joint have been obtained by (i) single pulse excitation method and (ii) Fourier analysis and compared.

Second order statistics of displacement, acceleration and contact force for periodically occurring random low joint have been obtained for fatigue analysis.

Second order statistics of response parameters have been calculated for continuous unevenness of the rail profile. The continuous unevenness has been treated

iv) (a) for a velocity less than 50 km/hr, interaction effect of only previous pulse is significant.

(b)For a velocity lies between 50 to 100 km/hr, effect of at least 3 pulses should be considered.

(c) For a velocity greater than 100 km/hr, effect of at least 5 pulses is necessary in summation. This interaction effect will again be insignificant as 5 rail lengths are welded for high velocity as found in practice.

v) For a design DLF of 1.77 and at a velocity of 60km/hr, probability of failure of the joint for one km length of track is as high, as 0.96. It increases as we decrease the value of design DLF.

(B) Response of vehicle to continuous unevenness shows that-

i) r.m.s. value of acceleration and contact force increases linearly at high velocity. Results very much depend upon chosen value of roughness constant.

(C) Response of vehicle to total unevenness shows that-

i) Expected value of total response process is same as that of expected value of response process due to descrete unevenness.

ii) r.m.s. value of the total response process increases with velocity.

7.3 FURTHER WORK

The work can be extended as follows:

(A) Vehicle Model:

i) Single degree of freedom model can be replaced by two degrees of freedom model with heave and pitch degrees of freedom.

ii) Interaction effect of successive passage of the two axles of vehicle can be considered.

iii) Parametric study of natural frequency (ω_o) and damping ratio (ζ) can also be considered. Non linearity in the spring and damper can also be taken into account.

(B) Track Model:

iv) Low joints have been assumed to have frozen profile. The rails can be assumed to be continuously supported on flexible foundation with the weight of the ties uniformly distributed and added to that of rail.

v) Length and depth of the joint (l,h) have been assumed to be uniformly distributed and linearly correlated. By collecting statistical data, actual probability distribution can be determined.

(C) Vehicle Motion:

vi) Constant velocity motion can be replaced by variable velocity run. Due to change in space time

relationship, the governing equation of motions in space coordinates will change.

vii) A detailed fatigue analysis of the vehicle and track components can be considered.

viii) Reliability based design of the track can be done.

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APPENDIX-I

REDUCTION OF EQUATION 2.2 TO A SYSTEM OF FIRST ORDER UNCOUPLED EQUATIONS

Consider equation 2.2 let

$$\left\{ q \right\} = \begin{pmatrix} \dot{Z} \\ Z \end{pmatrix} \qquad F(t) = \begin{pmatrix} 0 \\ 2\zeta\omega_0\dot{Y} + \omega_0^2 Y \end{pmatrix} \tag{A-1}$$

$$M = \begin{pmatrix} 0 & 1 \\ 1 & 2\zeta\omega_0 \end{pmatrix}, \qquad K = \begin{pmatrix} -1 & 0 \\ 0 & \omega_0^2 \end{pmatrix} \tag{A-2}$$

Equation 2.2 can now be written as

$$(M)\left\{ \ddot{q} \right\} + (K)\left\{ q \right\} = (F(t)) \tag{A-3}$$

The solution of homogenous part of equation is assumed to be of the form

$$\left\{ q \right\} = e^{\alpha t} \quad \text{A-4} \qquad \text{and} \qquad (D)(\phi) = \frac{1}{\alpha}(\phi) \tag{A-5}$$

Where

$$D = -(K)^{-1}(M) = \begin{pmatrix} -1 & 0 \\ 0 & \omega_0^2 \end{pmatrix}^{-1} \begin{pmatrix} 0 & 1 \\ 1 & 2\zeta\omega_0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 \\ -\frac{1}{\omega_0^2} & -\frac{2\zeta}{\omega_0} \end{pmatrix} \tag{A-6}$$

Where α and ϕ are the eigen values and eigen vectors of D.

The model column matrix is

$$(\Phi) = \left\{ \begin{matrix} \alpha_1\phi_1 & \alpha_2\phi_2 \\ \phi_1 & \phi_2 \end{matrix} \right\}$$

$$= \left\{ \begin{matrix} (\alpha\phi) \\ (\phi) \end{matrix} \right\} \tag{A-7}$$

Let $\{q(t)\} = [\Phi] \{x(t)\}$ A-8

Substituting A-8 into A-3 and using orthogonal property of eigen vector, yields a system of first order equations.

$$\dot{x}_i - \alpha_i x_i = F_i / M_i \qquad i = 1, 2$$ A-9

where $F_i = \phi_i \, (2 \zeta \omega_0 \dot{Y} + \omega_0^2 Y)$

$M_i = 2 \phi_i^2 (\alpha_i + \zeta \omega_0)$